

SOCIAL CONTAGION AND DIFFUSION: MODELING THE DIRECT AND INDIRECT PEER INFLUENCES ON REPEAT PURCHASE BY ONLINE GAME PLAYERS

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Abstract

Consumers' online purchase decisions can be affected by both direct and indirect peer influences. Literature has provided empirical evidence about both types of influence when the outcome is binary. However, there is no existing work about how these two types of peer influence impact repeat-purchase of consumers in a certain period of time. In this study, we want to fill such gap by investigating the interdependent repeat-purchase of online game players embedded in a social network. We build a new hierarchical Bayesian model that can support response variable following Poisson distribution and simultaneously include both types of peer influence, direct and indirect. Our empirical results show that both types of peer influence have significant impact on players' repeat-purchase decision but the influences may not have the same direction. Also, we find that the strength of the peer influences is associated with both network structure features and the strength of group-level pressure. The implication of our newly developed model includes first, helping online game providers develop effective targeting strategy to convert customers purchase more through peer influence, and second, comparing impact of different types of peer influences on repeat-purchase in social media platforms.

Keywords: Online games, Peer influences, Cohesion, Structural equivalence, Autoregression, Poisson regression, Hierarchical Bayesian framework.

1 INTRODUCTION

Literature shows that individuals' purchase decision can be affected by peer influence (Leenders 1997). For example, decisions about purchasing applications on Facebook website (Aral and Walker 2011), caller ringback tone on mobile phone platform (Ma et al. 2014), and iPhone 3G (Godinho de Matos et al. 2014) *etc.* can all be influenced by the neighbors in the social networks. However, there are two types of peer influence that provide very different explanation. The first type is direct influence, or cohesion. It explains that peer influence only comes from the direct (one-hop) neighbor because the impact has to be made through direct communication (Coleman et al. 1966; Rogers and Kincaid, 1981). The second type of influence is called as indirect influence, or structural equivalence. Such influence occurs when an individual imitates indirect (two-hop) neighbors with whom share common neighbors.

These two explanations about peer influence create two camps among scholars, and both camps have found empirical evidence to support their statements. Coleman et al. (1966), Rogers and Kincaid (1981), and Harkola and Greve (1995) demonstrate that decision about adoption is affected by direct influence, while Burt (1987), Strang and Tuma (1993), and Van den Bulte and Lilien (2001) find that adoption among individuals embedded in the social networks is driven by indirect influence. The debate between direct (cohesion) and indirect influences (structural equivalence) as the explanations of behavioral conformity remains ambiguous. Bowler et al. (2011) review papers that examine whether the social contagion effect is a result of relational adjacency (direct influence) or a similarity of position (structural equivalence). They propose that direct and indirect influences have independent and simultaneous impact on adoption behavior. So our manuscript aims to simultaneously examine the impact of these two peer influences on the counts of purchasing virtue goods by game players within a large in-game social network.

To our knowledge, our work is the first to study peer influence on individuals' repeat purchase. Traditional models for consumer repeat purchase, such as Pareto/NBD model (Schmittlein, Morrison, and Colombo 1987) and its hierarchical Bayes extension (Abe 2009), both ignore the interdependence of decisions by individuals within social networks. These two models address the heterogeneous preferences across individuals either by exogenous covariates or by independent and identical draws from a mixing distribution (Rossi and Allenby 2003). For literature studying the impact of peer influence on purchase decision (Yang and Allenby 2003; Zhang et al. 2013; Fang et al. 2013), only the circumstances of binary choice (purchase product or not) and continuous variable (amount of money) are investigated. Compared to those who purchase one expensive virtual goods but play much less and buy much less afterwards, game operator would be more interested in targeting players having high sustainability in the game. Thus, a prediction model supporting count data outcomes can be more useful than models supporting of binary or numeric outcomes.

The phenomenon we are interested in is peer influence on in-game purchase of online games. Our data is provided by a large online game operator in Asia. Online game industry has experienced rapid growth recently. The size of the global games market is forecasted to reach \$86.1 billion in 2015 (Internap.com 2015). The rapid growth of this market relies on two schemes, freemium business model and social networking embeddedness. Since most of the games adopt freemium business model, games are provided for free but advanced features, contents, and virtue goods are offered for a fee (Riggins 2003). The underlying assumption of the freemium model is that delivering a product for free can attract a large number of users and encourage participation, and a small fraction of participants will pay for the premium offer (Oestreicher-Singer & Zalmanson 2012). In order to broaden the influence of this small proportion of individuals, game designers usually embed social networking functionalities in online games. Evidence in theory and practice both indicates that peer influence among players within social networks facilitates the diffusion of players' willingness to pay for premium (Yee 2006; Joo et al. 2011). Thus, understanding peer influence on the repeat-purchase of virtual goods by the players is of great importance to online game operators.

In this study, we want to fill such gap by investigating the interdependent repeat purchase of online game players embedded in an in-game social network. We build a new hierarchical Bayesian model that can support response variable following Poisson distribution and simultaneously include

both types of peer influences, direct and indirect. Specifically, the direct and indirect peer influences are modeled as two network autocorrelation terms, which capture the correlated unobservable heterogeneity across individuals. Our empirical results show that both types of peer influences have significant impact on players' repeat purchase decision but the directions of the influences may be different. Such results can possibly be generated using a network autocorrelation model, the one our model belongs to. Furthermore, we find that the strength of the peer influences is associated with both network structure features, such as degree and betweenness centrality, and the strength of group-level pressure.

The remainder of this paper is organized as follows. Section 2 outlines related work on social influence in diffusion theory and models for consumer purchase. Section 3 provides the model specification and estimation procedures. Section 4 describes the data and illustrates the empirical results to analyze repeat purchase from players in a large online game. Section 5 offers a discussion of the results. Section 6 draws conclusions and future directions.

2 LITERATURE REVIEW

Purchasing the same product among a group of individuals embedded in a social network can be explained by the diffusion theory. Such theory explains how ideas, products, and practices spread among groups of people over time (Rogers 1962). Most recent diffusion models are proposed to understand the factors that lead individuals to adopt the behavior. This individual-level modeling approach examines consumer adoption behavior by considering individual heterogeneity in terms of attributes, behavior, and linkages in social networks. Two major research questions are supposed to be concerned in agent-based models: (i) how to identify peer influences on diffusion processes; (ii) how to model consumer preference interdependency on adoption decision.

Direct peer influence is the most common measure of network propagation, which captures social influence conveyed via information communication, persuasion, or direct pressure. Such peer influence between directly connected nodes is called social cohesion in the original study by Coleman et al. (1966). Existing literature on contagion caused by cohesion (direct influence) holds a basic view that people connected to social networks may induce his or her friends to behave in a similar way (Centola 2010). For instance, Wang and Chin (2011) studied the direct peer influence from pay users on a free user in freemium-based social networks. Results indicated that connection and interaction with pay users significantly increased the probability of being a pay user. Literature on direct peer influence consistently shows a positive correlation between the adoption of focal individual and her social neighbors. In our context, it indicates that players connecting to friends who have high value in repeat purchase will also purchase more. Thus we propose following hypothesis:

Hypothesis 1. The number of repeat purchase of the focal player is positively correlated with the number of repeat purchase from direct friends.

The location in social network may also affect individual's adoption of information, opinions, and behaviors. Marsden and Friedkin (1993) extended the concept of social proximity by allowing the social correlation among the individuals who are not direct adjacency. Structural equivalence (indirect peer influence) is a fundamental characteristic to conceptualize positional similarity in network (Lorrain & White 1971). Such generalization of adjacency permits that individuals are proximate to the extent of profile similarity of "equivalent environments" in which members are tied to the same types of individuals (Borgatti & Everett 1992). Specifically, if people occupy the same position in social structure and share common friends, they likely exhibit similar opinions or behaviors because "they are identically positioned in the flow of influential communication and use each other as a frame of judgment reference, even without any communications" (Burt 1987). It provides a competing explanation for social contagion by addressing the homogeneity between indirectly connected entities, instead of direct influence (cohesion). But the direction of such indirect peer influence accompanies with conflicting researches (Akerlof 1997; Lerner and Tirole 2005; Singh and Phelps 2013). Empirical result by Singh and Phelps (2013) shows that structural equivalent project managers are likely to choose the same open source software license. Nevertheless, Mizuchi (1993) and Zhang et al. (2013) find negative indirect peer influence on political behavior among competing firms and on

subscription decision on ‘Caller Ring Back Tones’ services. They point out that negative indirect peer influence may be observed when individuals intend to differentiate themselves from individuals who share common friends with them. In terms of game context, players who share common friends in the game have motivations to compete with each other to attract more in-game friends or teammates. Based on the mixing results of the effect of indirect peer influence, we accordingly make alternative hypotheses that:

Hypothesis 2a. The number of repeat purchase of the focal player is positively correlated with the number of repeat purchase behavior of indirect neighbors sharing common neighbors.

Hypothesis 2b. The number of repeat purchase of the focal player is negatively correlated with the number of repeat purchase behavior of indirect neighbors sharing common neighbors.

In addition to these two proximity measures (cohesion and structural equivalence), centrality, another important network properties, can also be used to reflect social influence. There are at least ten centrality measures in literature. Chen et al. (2012) comment that degree centrality is simple but of little relevance to identify opinion leaders, while betweenness centrality and closeness centrality are better detectors but incapable to be applied in large-scale networks due to computation burden. Unlike cohesion and structural equivalence, centrality measures are node-specific properties which cannot be used to describe dyadic relationship (Narayan and Yang 2006). Therefore, degree, betweenness, and closeness often act as control or instrumental variables in regression equations (Tucker 2004; Katona, Zubcsek, & Sarvary 2011; Peng & Dey 2013).

The literature has studied virtual product consumption mainly from three aspects: attitudes and intention of consumers, attributes and values of products, and the dynamics of the environment. Most of them have been conceptual work using structural equation modeling. For instance, Chung (2005) investigated the impact of individual beliefs and attitudes on money and frequencies regarding purchasing virtual products by applying the Technology Acceptance Model and the Theory of Reasoned Action. Product attributes are regarded as purchase drivers in the context of heterogeneous product consumption decision (Kim, Gupta, & Koh 2011). The repeat purchase of virtual items in online games differ from the other products in the real world in the way that players cannot buy virtual items directly with real money. Players have to purchase the in-game money first with the real money and then obtain virtual items through the in-game currency. In our study, we focus on the repeat purchase of in-game currency at the first step and do not care about how the players use the in-game money. Therefore, attributes and values of products are not considered in our model. Environmental dynamics address the role of peer influence and herding effect on purchase behavior (Animesh et al. 2011; Chen 2008).

In order to examine the effects of direct and indirect peer influences, we have to control other factors that may affect repeat purchase behavior, such as individual attributes including demographical and behavioral characteristics (Katona et al. 2011). According to the existing literature, peer influence may affect individuals’ purchase decision in two different ways, observed behavior imitation (Lin 2010) or latent preference correlation (Yang & Allenby 2003). Leenders (2002) used spatial autoregressive model (LeSage 1997) to describe the correlation of observed behavior among peers. In this model, y appears as both the dependent and independent variable.

$$\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I}_n)$$

As to latent preference correlation, spatial error model is applied using network data (Yang & Allenby 2003; Zhang et al. 2014), in which the interdependent preference $\boldsymbol{\theta}$ is unobservable and represented as the network autocorrelation term.

$$\begin{aligned} \mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\theta} + \boldsymbol{\varepsilon} \\ \boldsymbol{\theta} &= \rho \mathbf{W}\boldsymbol{\theta} + \mathbf{u} \end{aligned}$$

In our context, the in-game purchase behavior cannot be observed by peers. Therefore, the peer influence in online games is supposed to be reflected by interdependent latent preferences among individuals. Although Zhang et al.’s multi-network autoprobit (mNAP) model (2014) is capable to

compare multiple types of peer influence, the response variable has to be binary, while we want to study repeat purchase of players in online games. So we create a new hierarchical Bayesian model that can support the analysis of repeat purchase made by players.

3 MODEL

So far existing models are mainly for continuous or binary type of response variables (Coleman et al. 1966; Leenders 2002; Zhang et al. 2013; Fang et al. 2013). There is no model available to study peer influence on individuals' repeat purchase behavior, much less to accommodate multiple types of influences. To our knowledge, we are the first to design a Poisson regression model with two network autocorrelation terms that captures both direct and indirect peer influences among individuals. The prior distribution and parameter estimation for posterior is provided in this section. The model is implemented using Markov chain Monte Carlo (MCMC) approach. The detailed steps of posterior distribution derivations can be found in Appendix A.

We observed a sequence of transactions made by a set of individuals ($i = 1, \dots, n$). We assume that the count of individual purchase follows a Poisson distribution with purchase rate parameter, which varies across individuals. Accordingly, we model the number of transactions made by all players, \mathbf{y} , using a Poisson regression that specifies the relationship between the purchase rate vector λ and predictor matrix \mathbf{X} (Cameron & Trivedi 2013). The predictors \mathbf{X} are defined by players' individual attributes and guild-level attributes. Individual attributes include levels, online duration, login count, and count of quests completed *etc.* Guild-level attributes include number of paid members in the guild, guild level, guild wealth and a leader flag *etc.* We also add degree centrality as control variables in the regression equation. $\boldsymbol{\beta}$ is the correspondent coefficient for \mathbf{X} .

$$\begin{aligned} \mathbf{y} &\sim \text{Poisson}(\boldsymbol{\lambda}) \\ \log(\boldsymbol{\lambda}) &= \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\theta} + \boldsymbol{\varepsilon} \\ \boldsymbol{\theta} &= \rho_1 \mathbf{W}_1 \boldsymbol{\theta} + \rho_2 \mathbf{W}_2 \boldsymbol{\theta} + \mathbf{u} \\ \boldsymbol{\varepsilon} &\sim N(\mathbf{0}, \mathbf{I}) \\ \mathbf{u} &\sim N(\mathbf{0}, \sigma^2 \mathbf{I}) \end{aligned}$$

$\boldsymbol{\theta}$ is a vector of autoregressive parameters which represents the interdependent latent preference players' repeat purchase, and can be described by the summation of two network autocorrelation terms $\rho_1 \mathbf{W}_1 \boldsymbol{\theta}_1 + \rho_2 \mathbf{W}_2 \boldsymbol{\theta}_2$. The scalar ρ_1 and ρ_2 are coefficients for $\mathbf{W}\boldsymbol{\theta}$. Direct (cohesion) and indirect influence (structural equivalence) are represented by matrix \mathbf{W}_1 and matrix \mathbf{W}_2 respectively. Thus the existence of peer influence will be simply presented by the significant level of ρ_1 and ρ_2 . We define that the diagonal elements of the two matrices are equal to zero, so no self loop is permitted. The elements a_{ij} in matrix \mathbf{W}_1 equal one if i and j are friends and zero otherwise. We specify matrix \mathbf{W}_2 as the power of structural equivalence between any two players, which is calculated by the inverse of Euclidean distance d_{ij} plus one (Zhang et al. 2013). The higher value of distance (d_{ij}), the less structural equivalence between the two nodes.

$$\begin{aligned} \mathbf{W}_1 &= \{a_{ij}\}, \text{ where } a_{ij} \in \{0, 1\} \\ \mathbf{W}_2 &= \left\{ \frac{1}{d_{ij} + 1} \right\}, \text{ where } d_{ij} = \sum_{z \in V \setminus \{i, j\}} (a_{iz} - a_{jz})^2 \end{aligned}$$

We use Bayesian method to solve the Poisson regression model with network autoregression terms. It requires specification of prior distributions for each parameters. The details are shown in Table 1. We define a hybrid MCMC sampler scheme for this model. The basic idea in Bayesian estimation is to proceed a Markov chain by generating samples from the set of posterior distributions successively, that is, $p(\boldsymbol{\lambda}|\mathbf{y}, \boldsymbol{\beta}, \boldsymbol{\theta})$, $p(\boldsymbol{\beta}|\boldsymbol{\lambda}, \boldsymbol{\theta})$, $p(\boldsymbol{\theta}|\boldsymbol{\rho}, \sigma^2, \boldsymbol{\lambda}, \boldsymbol{\beta})$, $p(\sigma^2|\boldsymbol{\rho}, \boldsymbol{\theta})$, and $p(\boldsymbol{\rho}|\sigma^2, \boldsymbol{\theta})$. Since the model settings of parameter $\boldsymbol{\lambda}$ and $\boldsymbol{\rho}$ are not conditionally conjugate, the sampling for the posterior distribution $p(\boldsymbol{\lambda}|\mathbf{y}, \boldsymbol{\beta}, \boldsymbol{\theta})$ and $p(\boldsymbol{\rho}|\sigma^2, \boldsymbol{\theta})$ are realized by using metropolis-hasting algorithm that is an

adaptation of a random walk that uses an acceptance rule to converge to the specified target distribution. The detailed description of MCMC estimation implementation is attached in Appendix A.

Parameter	Density	Draw Type
β	Normal($\boldsymbol{\mu}_\beta, \mathbf{V}_\beta$)	Parallel
ρ_1, ρ_2	Normal($\boldsymbol{\mu}_\rho, \mathbf{V}_\rho$)	Sequential Metropolis Step
σ^2	InvGamma(a, b)	Single

Table 1. Prior distribution specification (Note. The normal distribution of ρ_1, ρ_2 should be restricted to ensure that the matrix $(I - \rho_1 \mathbf{W}_1 - \rho_2 \mathbf{W}_2)$ is invertible.)

4 RESULTS

Our data is obtained from a large online game operator in Asia. It is a free-to-play fantasy massively multiplayer online role-playing games (MMORPG) which was released in 2007. Our dataset consists of the transaction records of 12,220 players over a 12-week period in the same game. Considering the sparsity of individual transaction activities, we aggregate the number of transactions made by players using a two-week time window. The data of personal attributes, guild-level features, and social network structure are captured every two weeks as well, thus ending up with six sequential profiles ($t=1, 2, \dots, 6$). In each time period, we defined the network connections using in-game built friendship relations among players. The visualization structure of social network at $t=1$ is demonstrated in Figure 1. We can easily recognize that the large network can be decomposed into four sub-units or communities. In order to capture the unique peer influence pattern in each self-contained community, we apply the fast greedy community detection algorithm (Clauset et al. 2004) to partition the network into four communities of densely connected nodes ($c=1, 2, 3, 4$), in which the nodes belonging to different communities are sparsely located (Blondel et al. 2008).

We estimated the peer influences in each community using hierarchical Bayesian approach and implement MCMC methods. We ran several long chains and monitor the chains for convergence. The first 2,000 draws were discarded as “burn-in” period (Plummer et al. 2005), so we ran an additional 8,000 draws. The estimation sequence was thinned by keeping every 20th simulation to avoid autocorrelation problems among MCMC iterations. In this way, 400 valid estimation results are generated for each of the hyper parameters. In order to compare the overall effect of social influences in the four different communities, we accumulate the estimation results across the six time periods using a random effect meta-analysis approach (Borenstein et al. 2011). The posterior coefficient estimates across the six time periods are assumed to be a random sample of the relevant distribution of true effects, and the combined effect are estimated as the mean of the population of true effects. Therefore, in assigning weights to studies, both the sampling error within and between studies should be dealt with. The meta-analysis results are presented in Table 2.

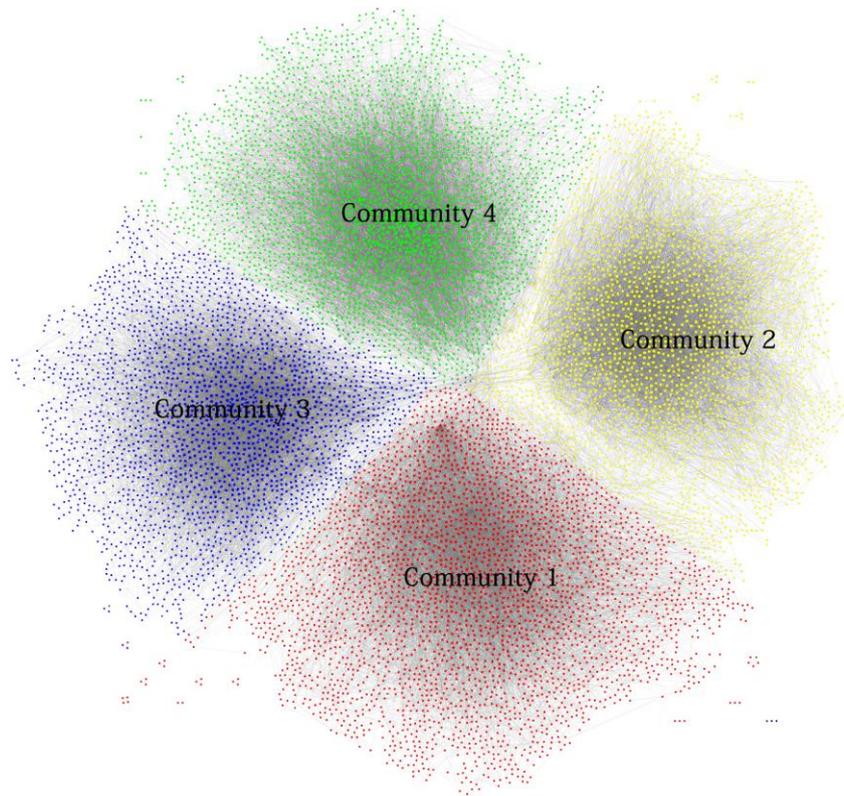


Figure 1. Plot of in-game social network structure at $t=1$

Coefficient	Community 1	Community 2	Community 3	Community 4
Achievement (β_1)	0.22** (0.034)	0.23** (0.037)	0.26** (0.042)	0.23** (0.039)
Level (β_2)	-0.013** (0.0041)	-0.0059 (0.0043)	-0.0014 (0.0033)	-0.010** (0.0032)
Duration (β_3)	0.20** (0.026)	0.15** (0.029)	0.15** (0.025)	0.049* (0.022)
Login frequency (β_4)	0.29** (0.034)	0.28** (0.040)	0.28** (0.047)	0.37** (0.045)
Quest frequency (β_5)	-0.089** (0.024)	-0.089** (0.027)	-0.11** (0.025)	-0.078** (0.022)
Guild paid member (β_6)	0.0032* (0.0014)	0.0027 (0.0015)	-0.0017 (0.0020)	0.0032* (0.0016)
Guild level (β_7)	0.018 (0.022)	0.15 (0.10)	-0.13 (0.13)	0.17** (0.057)
Guild money (β_8)	0.00027 (0.0040)	-0.0074 (0.0057)	0.0043 (0.0023)	0.024** (0.0062)
Leader or not (β_9)	-0.063 (0.050)	0.015 (0.055)	-0.051 (0.051)	-0.034 (0.048)
Degree (β_{10})	0.0092** (0.0017)	0.019** (0.0020)	0.012** (0.0020)	0.011** (0.0021)
Direct influence (ρ_1)	0.038** (0.0032)	0.031** (0.0046)	0.017** (0.0056)	0.022** (0.0056)
Indirect influence (ρ_2)	-0.098** (0.022)	-0.015 (0.0082)	-0.0066 (0.0058)	-0.059** (0.013)

** : $p < 0.01$; * : $p < 0.05$

Table 2. Estimated coefficients from our Bayesian model across all time periods

Peer influences which is indicated by parameters ρ_1 and ρ_2 are the key elements of interest in our model. As we have defined, ρ_1 represents the direct peer influence, while ρ_2 addresses the indirect peer influence. The overall effects (in Table 2) of direct influence in all the four communities are significantly positive, which supports our hypothesis 1 that the behavior of neighboring players are positively correlated. Specifically, direct influence in communities 1 and 2 is stronger than in communities 3 and 4. However, indirect influence has negative effect on the repeat purchase and it is only significant in communities 1 and 4, confirming our hypothesis 2b. In terms of the estimates in each time period (as shown in Table 3), community 1 is a strong social network embedded group, in which both direct and indirect influences have consistently significant effects. Community 4 is constrained social network embedded group, in which direct and indirect peer influences are not always significant and with smaller magnitudes. Community 2 is a strong friendship embedded group, in which only direct peer influence has consistently significant effects. Communities 3 can be regarded as weak social network embedded groups, in which direct and indirect influences take insignificant effects across time periods.

	Community 1		Community 2		Community 3		Community 4	
	ρ_1	ρ_2	ρ_1	ρ_2	ρ_1	ρ_2	ρ_1	ρ_2
t=1	0.037** (0.0085)	-0.076* (0.035)	0.044** (0.0095)	-0.032 (0.035)	0.019 (0.013)	-0.007 (0.011)	0.031** (0.013)	-0.079** (0.032)
t=2	0.034** (0.0087)	-0.14** (0.093)	0.031** (0.010)	-0.011 (0.017)	0.017 (0.013)	-0.0043 (0.0071)	0.029** (0.010)	-0.019 (0.024)
t=3	0.042** (0.0054)	-0.087* (0.045)	0.012 (0.013)	-0.013 (0.025)	0.021 (0.0018)	-0.11 (0.099)	0.011 (0.012)	-0.035 (0.031)
t=4	0.041** (0.0040)	-0.13** (0.074)	0.016 (0.014)	-0.014 (0.028)	0.016 (0.016)	-0.091 (0.086)	0.018 (0.013)	-0.056* (0.029)
t=5	0.031** (0.0084)	-0.28** (0.10)	0.024** (0.012)	-0.011 (0.013)	0.0039 (0.013)	-0.14 (0.14)	0.017* (0.010)	-0.14** (0.047)
t=6	0.033** (0.0069)	-0.084** (0.052)	0.039** (0.0076)	-0.029 (0.024)	0.013 (0.0089)	-0.058 (0.051)	0.024** (0.010)	-0.17** (0.049)

** : $p < 0.05$; * : $p < 0.1$

Table 3. Peer influence estimation results for every two weeks

The sizes of these four communities are similar, so their community level network features, such as density and diameter ($F=1.64$, $p=0.212$). However, social influences play quite different roles in the four communities as illustrated above. It has been proved that network structure associates with the strength of peer influence. For example, the higher the density of a network, the stronger the peer influence among individuals embedded in the network. We conducted ANOVA analysis on network structure features at individual level, including degree and betweenness centrality, across the four communities. The ANOVA analysis of 6 two-week periods consistently demonstrated that the average degree of players in community 1 is significantly smaller than the other 3 communities and the average betweenness centrality of players in community 1 is significantly larger than the others. The details about ANOVA analysis are shown in Appendix D. When only control for average degree centrality in the community and average betweenness centrality, direct peer influence in community 4 is weaker than that in community 2 cannot be explained. However, when we add control for guild-level attributes, the direct influence can be explained by group-level influence. The results suggest that both group-level and network structure attributes can affect the strength of both types of peer influences.

Another property of our model is to predict the purchase behavior of players in future time period. According to the memoryless feature of Poisson process, individual purchase behavior during a certain period of time in future can be easily predicted by a random sample from the Poisson

distribution with known purchase rate λ . We applied a pattern-matching scheme based on the quantile-quantile plot (Q-Q plot) to examine the performance of our model focusing on the quality of predictions of individual-level transactions in the forecast period (the next two weeks). In the following Q-Q plot (Figure 2), the quantiles of data set about predicted repeat purchase based on estimates of λ at time $t=1$ are plotted against the corresponding quantiles of the actual repeat purchase at time $t=2$. The red line of dashes is the 45 degree reference line. If the two data sets come from a population with the same distribution, the points (with the plot symbol '+') should fall approximately along this reference line (Li et al. 1999). According to the images of four communities, it is difficult to tell which one has a smaller departure from the reference line. The p-value of chi-square test from the resulting Q-Q plots is then used as the quantitative measure of similarity among the compared images of four communities (Tsai & Yang 2005). A small p-value approximate to zero suggests a strong heterogeneity between two compared samples. As it indicates in Table 4, the prediction results in all communities have a strong similarity with the actual behavior patterns, especially the performance of our model in community 1 and community 3 are even better (close to unit).

	Community 1	Community 2	Community 3	Community 4
p-value	0.958	0.878	0.976	0.903

Table 4. Comparison of similarity measures of Q-Q plots of the four communities

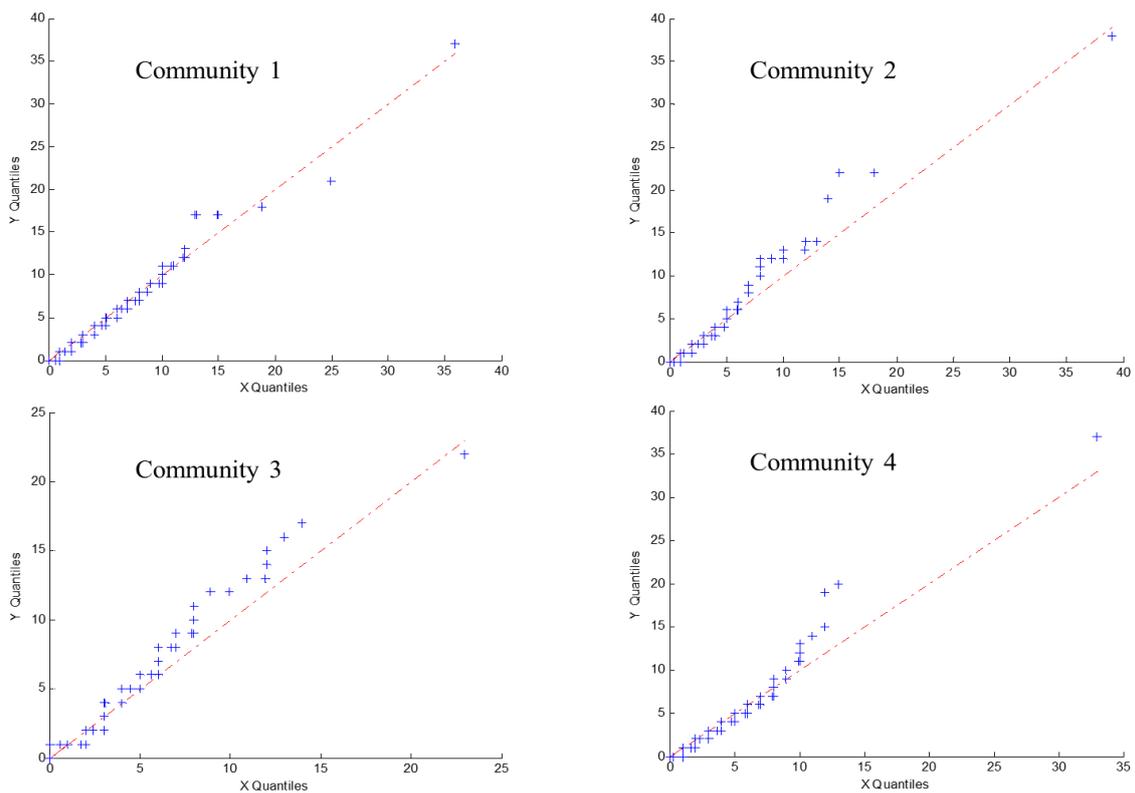


Figure 2. Q-Q plot comparing the actual repeat purchase (x-axis) at $t=2$ with the simulated random sample (y-axis) generated from Poisson distribution with estimated purchase rate λ at $t=1$

Q-Q plot approximating to 45 degree line only guarantees that prediction results and actual data come from populations with the same distribution. We need to make comparative analyses of the prediction performance at individual-level. We benchmark the performance of our proposed model against a simple Poisson Regression model described in Table 5. For model fit and comparison, we use the deviance information criterion (DIC) introduced by Spiegelhalter et al.(2002), which is the most common method of assessing the goodness of fit in Bayesian inference. The model with the

smallest DIC is estimated to be the model that best predict a replicate dataset which has the same structure as the currently observed. As Table 6 suggests, the proposed model provides a significantly better fit to the data than the baseline model. Besides, we predict that the number of transactions made by individuals in time period $t+1$ on the basis of MCMC estimates. We conducted 5 evaluations for each community to compare the prediction performance of the proposed model and a simple Poisson Regression baseline model, with t ranging from 2 to 6. To diagnose the variation in the errors in the count data forecasts, mean absolute deviation (MAD) and rooted mean squared error (rMSE) are used together (Legates and McCabe 1999). Lower values are better. Table 6 shows the prediction accuracy of the proposed model and the baseline model. In all evaluations, our proposed model outperforms the baseline model. The superiority of the proposed method over the baseline method can be attributed to the consideration of hidden correlated structure of the error term caused by peer influences.

Model	Specification	Fit (DIC)
baseline model (BM)	$Y \sim \text{Poisson}(\lambda)$ $\log(\lambda) = X\beta + \varepsilon$	8636.7
proposed model (PM)	$Y \sim \text{Poisson}(\lambda)$ $\log(\lambda) = X\beta + \theta + \varepsilon$ $\theta = \rho_1 W_1 \theta + \rho_2 W_2 \theta + u$	6878.6

Table 5. Model Fit Comparison

Evaluation time (t+1)	Criteria	Community 1		Community 2		Community 3		Community 4	
		PM	BM	PM	BM	PM	BM	PM	BM
t=2	rMSE	3.145	3.458	2.732	2.924	2.155	2.451	2.749	2.851
	MAD	1.672	1.972	1.097	1.251	0.915	1.095	1.181	1.303
t=3	rMSE	3.116	3.221	2.475	3.256	2.488	2.881	2.809	3.032
	MAD	1.524	1.739	0.982	1.423	1.057	1.139	1.105	1.282
t=4	rMSE	2.768	2.976	2.316	2.739	2.329	2.581	3.201	3.489
	MAD	1.247	1.468	0.857	1.198	1.053	1.005	1.281	1.300
t=5	rMSE	2.428	2.797	2.186	2.460	2.198	2.603	3.017	3.330
	MAD	1.071	1.287	0.789	0.985	0.833	1.335	1.003	1.179
t=6	rMSE	2.088	2.776	2.020	2.261	2.083	2.227	2.143	2.378
	MAD	0.864	1.195	0.713	0.844	0.817	1.081	0.843	0.955
mean	rMSE	2.709	3.046	2.346	2.728	2.251	2.549	2.784	3.016
	MAD	1.276	1.532	0.888	1.140	0.935	1.131	1.083	1.204

Table 6. Prediction accuracy analyses of proposed and baseline models

5 DISCUSSION

We make a methodology contribution by creating a new Poisson regression model with two network autocorrelation terms that represent the impact of both direct and indirect peer influences on repeat purchase behavior of online game players. The augmented-error model contributes to nonzero covariance of purchase rate λ , which sheds light on two possible sources of the interdependent preferences from relational and positional social influence perspectives. We use an alternative model specification to investigate the purchase behavior and find that the standard Poisson regression model

is inferior to an autoregressive specification. The autocorrelation methodology is seen as a useful approach measuring the degree of overall network correlation among individuals beyond that captured by the covariates \mathbf{X} . The significance level and the sign of coefficients ρ_1 and ρ_2 both provide understanding about the existence of peer influences.

Our study expands IS literature by studying the diffusion of repeat purchase for virtual goods in online games. The frequency analysis of behavior is able to detect the non-progressive phenomena, in which an individual may start using or purchasing a product and then possibly stop at some point. The first adoption behavior does not guarantee that the individual will continue using or purchasing the product in the future. Using a count type data, we are able to monitor the fluctuation of players' repeat purchase behavior. In addition, our model can be applied to study the frequency of login as well as content creation and consumption in social media, the frequency of knowledge generation by individuals in enterprise collaboration platform, etc., which will provide broader insights for diffusion theory.

Our work makes theoretical contribution by reconciling the cohesion (direct influence) and structural equivalence (indirect influence) debate. We measure the extent to which behavioral conformity is a consequence of either direct ties between individuals or indirect ties of individuals with similar friends. And we find that both of the two peer influences may exert significant influence on players' purchase behavior but the direction of effect varies. The positive correlation of heterogeneity among direct connected players is in accord with the expectation. However, indirect peer influence (structural equivalence) is intermittently significant with its sign in the opposite from expected direction. Previous literature on structural equivalence (indirect influence) provides mixed evidence as well. Ibarra and Andrews (1993) believed that indirectly connected individuals sharing common friends will have similar behavior due to shared experiences, common socialization, similar role demands, and similar expectation from others. On the contrary, the empirical work of Mizuchi (1993) and Zhang et al. (2013) revealed negative indirect peer influence on political behavior among competing firms and on subscription decision on 'Caller Ring Back Tones' services. According to Shah (2000), indirect connected individuals with common friends share similar pattern of relationships, thus, an inherent rivalry exists, which means an individual can be substituted by the other. Purchasing behavior of player usually occur as a remedial action for the lack of experience in games. Similar with the strategy of competing firms, players' investment decisions will be protected from imitation by indirect connected individuals sharing common friends. So insignificant indirect peer influence and even inverse behavior mimicry among indirect connected players sharing common friends are observed in our studies.

Another crucial finding is that the strength of direct and indirect peer influences vary across different subnetworks. We have verified that our data is collected from four servers, and each server has one independent copy of the game. The four communities of paid players belong to four parallel virtual worlds built on different servers. On the surface, the communities are similar in size, besides, the density and diameter of the networks do not indicate large differences. But the magnitude and significance of direct and indirect peer influences vary across the four communities. We proposed three relevant associations. First, the average degree centrality of nodes in network is negative related with the strength of direct peer influence (cohesion). As it is defined, degree is the number of first-order friends the individual has. Within a connected network, the lower average degree is, the more efficient information diffuses. Thus, the direct peer influence through each social tie is more likely to take strong effects. Similar results have been found by Ugander et al. (2012) that the size of neighborhood generally has negative effect on social contagion, when structural diversity is controlled. Second, the average betweenness centrality is positive associated with the strength of indirect influence (structural equivalence). Other than degree, we use betweenness centrality to measure the influence power from social positions. Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes, which addresses the important role of common friends in the concept of structural equivalence (indirect influence). Thus, high betweenness in average indicates indirect peer influence is more likely to make strong significant influence. However, recognized differences in network structure features above cannot fully explain the abnormal estimation results for community 4 in which the strength of direct peer influence does

not rank the second place as expected. Then we reveal the third association between social influences and group-level influences in four communities. We find that if the group-level influence on purchase is strong, the direct peer influence will be weakened. It indicates that the external pressure from the guild may reduce the behavioral dependency between in-game friends. Previous studies on the impact of diversified relationships in organization on behavioral processes have examined the substitution effect when blended relationships exist (Ragins 1997). According to the first association, community 4 has a relatively low degree centrality in average should have a strong direct peer influence. But due to the strong effect of guild influence, direct peer influence in community 4 is weakened.

Our work can also provide implications to practitioners, especially to online game operators. First, our model has an effective capability to predict players' future transactions based on the estimated value of purchase rate in Poisson distribution. In addition, the Bayesian estimate of purchase rate for each player helps online game providers and operators develop effective targeting strategy to convert customers purchase more through peer influence. Second, the regression part provides a comprehensive set of key factors underlying players' repeat-purchase behavior, including individual attributes, group-level influence, and peer influences. Our findings suggest that managers should take advantage of positive peer influence through direct social connections and regulate the information asymmetry between indirect connected players sharing common friends. Advertising and promotion efforts should be invested on influential players and communities with effective network structures.

6 CONCLUSION

We design a new hierarchical Bayesian model to identify the key factors that motivate online game players' repeat-purchase behavior on the basis of personal and network structure attributes. We compare the impact of both direct and indirect peer influences on the repeat purchase of in-game virtual good. Both types of influences are modelled as network autocorrelation term to capture the interdependent purchase behaviour. Empirical results indicate that both direct and indirect peer influences can have impact on repeat purchase simultaneously. The extent to which the behavior of direct or indirect connected players sharing common friends are correlated is associated with network structure features as well as the strength of group-level pressure. Properly identifying such endogenous influence structure provides substantial benefits to effective future transactions prediction.

Our model could be extended in several directions. First, we want our model to support longitudinal data. A fixed effects term accounting for individual level unobserved heterogeneity across time will be added. By using a longitudinal model, we are able to identify the change in the repeat purchase behavior that is caused by the change in players' personal attributes and peer influences. Thus the causality of peer influence on repeat purchase can be achieved. Second, our model inherits the stationary property of Poisson process, which assumes players are active and have equivalent expected activity level across time periods. However, consumer defection is a theoretical possibility, although our 12-week observation data focusing on paid players suggests that the dropout rate, if any, is small. In order to evaluate the likelihood of consumer defection in a long-term analysis, consumer behavior should be characterized by another exponential lifetime duration function (with a certain dropout rate) together with Poisson purchase process. These applications and extensions will contribute to a better understanding of consumer repeated purchase behavior.

APPENDIX A

MCMC estimation steps for Poisson regression model with network autoregression term

In the convenience of notation, we use $\boldsymbol{\eta}$ to represent $\log(\boldsymbol{\lambda})$. Thus, the model is specified as follows:

$$\begin{aligned}\mathbf{y} &\sim \text{Poisson}(\exp(\boldsymbol{\eta})) \\ \boldsymbol{\eta} &= \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\theta} + \boldsymbol{\varepsilon} \\ \boldsymbol{\beta} &\sim N(\boldsymbol{\beta}_0, \mathbf{D}) \\ \boldsymbol{\theta} &= \rho_1 \mathbf{W}_1 \boldsymbol{\theta} + \rho_2 \mathbf{W}_2 \boldsymbol{\theta} + \mathbf{u} \\ \mathbf{u} &\sim N(0, \sigma^2 \mathbf{I}_n) \\ \boldsymbol{\varepsilon} &\sim N(0, \mathbf{I}_n) \\ \sigma^2 &\sim IG(s_0, q_0)\end{aligned}$$

The posterior distribution of $\boldsymbol{\eta}$ conditional on the data is:

$$p(\boldsymbol{\eta} | \boldsymbol{\beta}, \sigma^2, \boldsymbol{\theta}, \mathbf{X}, \mathbf{y}) \propto p(\boldsymbol{\eta} | \boldsymbol{\beta}, \boldsymbol{\theta}, \mathbf{X}) p(\mathbf{y} | \boldsymbol{\eta}) \propto \prod_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(\eta_i - \sum_{k=1}^K x_{ik} \beta_k - \theta_i)^2}{2}\right) \frac{\exp(\eta_i)^{y_i} \exp(-\exp(\eta_i))}{y_i!}$$

Draw each η_i with Metropolis-Hasting algorithm

$$\begin{aligned}\eta_i^{new} &= \eta_i^{old} + \Delta_i \\ \Delta_i &\sim N(0, 1)\end{aligned}$$

The acceptance probability is given by

$$\frac{\exp(-\exp(\eta_i^{new}) - 0.5 * (\eta_i^{new} - \sum_{k=1}^K x_{ik} \beta_k - \theta_i)^2 + y_i \eta_i^{new})}{\exp(-\exp(\eta_i^{old}) - 0.5 * (\eta_i^{old} - \sum_{k=1}^K x_{ik} \beta_k - \theta_i)^2 + y_i \eta_i^{old})}$$

Draw $\boldsymbol{\beta}$, the prior distribution is $\boldsymbol{\beta} \sim N(\boldsymbol{\beta}_0, \mathbf{D})$, $\boldsymbol{\beta}_0 = (0, \dots, 0)$, $\mathbf{D} = 400\mathbf{I}_n$

$$\begin{aligned}\boldsymbol{\beta} | \sigma^2, \boldsymbol{\eta}, \boldsymbol{\theta} &\propto N(\boldsymbol{\mu}_\beta, \mathbf{V}_\beta) \\ \boldsymbol{\mu}_\beta &= \mathbf{V}_\beta (\mathbf{X}^T (\boldsymbol{\eta} - \boldsymbol{\theta}) + \mathbf{D}^{-1} \boldsymbol{\beta}_0) \\ \mathbf{V}_\beta &= (\mathbf{D}^{-1} + \mathbf{X}^T \mathbf{X})^{-1}\end{aligned}$$

Draw $\boldsymbol{\theta}$, the prior distribution is $\boldsymbol{\theta} \sim N(0, \mathbf{1})$

$$\begin{aligned}\boldsymbol{\theta} | \rho_1, \rho_2, \sigma^2, \boldsymbol{\eta}, \boldsymbol{\beta} &\sim N(\boldsymbol{\mu}_\theta, \mathbf{V}_\theta) \\ \mathbf{B}(\rho_1, \rho_2) &= (\mathbf{I} - \rho_1 \mathbf{W}_1 - \rho_2 \mathbf{W}_2) \\ \boldsymbol{\mu}_\theta &= \boldsymbol{\Sigma}_\theta (\boldsymbol{\eta} - \mathbf{X}\boldsymbol{\beta}) \\ \boldsymbol{\Sigma}_\theta &= (\mathbf{I}^{-1} + \sigma^{-2} \mathbf{B}^T \mathbf{B})^{-1}\end{aligned}$$

Draw σ^2 , the prior distribution is $\sigma^2 \sim IG(s_0, q_0)$, $s_0 = 5$, $q_0 = 10$

$$\begin{aligned}\sigma^2 | \rho_1, \rho_2, \boldsymbol{\theta} &\sim IG(a, b) \\ a &= s_0 + \frac{n}{2}, \quad b = q_0 + \frac{\boldsymbol{\theta}^T \mathbf{B}^T \mathbf{B} \boldsymbol{\theta}}{2}\end{aligned}$$

a, b are shape and rate parameters of gamma distribution $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$

Draw ρ_1, ρ_2 with Metropolis-Hasting algorithm

$$\rho_i^{new} = \rho_i^{old} + \Delta_i, \quad \Delta_i \sim N(0, 0.005^2)$$

The acceptance probability is given by

$$\min\left(\frac{|\mathbf{B}(\rho_i^{new})| \exp\left(-\frac{1}{2\sigma^2} \boldsymbol{\theta}^T \mathbf{B}(\rho_i^{new})^T \mathbf{B}(\rho_i^{new}) \boldsymbol{\theta}\right)}{|\mathbf{B}(\rho_i^{old})| \exp\left(-\frac{1}{2\sigma^2} \boldsymbol{\theta}^T \mathbf{B}(\rho_i^{old})^T \mathbf{B}(\rho_i^{old}) \boldsymbol{\theta}\right)}, 1\right)$$

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