# AN ONLINE ALGORITHM FOR PROGRAMMATIC ADVERTISEMENT PLACEMENT IN SUPPLY SIDE PLATFORM OF MOBILE ADVERTISEMENT

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## Abstract

*Smartphone applications are emerging as popular media for promoting one's products* through in-app *advertisements. Today, there are a number of organizations, known as supply-side-platforms (SSP), who aggregate and auction these ad-spaces from different suppliers/publishers. Advertisers (or their intermediaries) place bid for these spaces based on different relevance criteria (e.g., the location and device of the app-user, the app's IAB category etc.), the impression value, clickthrough* value, and the *conversion value. After the received ads are filtered based on relevance, the SSP is often still faced with a number of options for ad-placement, each having different revenues owing to differences in clickthrough rates etc. Moreover, the SSP has to decide on the ad-placement in real-time. In this paper, we consider the SSP's ad-placement problem in the aforementioned situation. We propose an optimization model to maximize the SSP's revenues. Based on computational experience with this model, we develop a rule-based online algorithm that appears to be viable as a real-time solution.*

*Keywords: Supply Side Platform, Demand Side Platform, Real Time Bidding, Online Algorithm.*

# 1 INTRODUCTION

The proliferation of affordable smartphones (Android, iOS and Windows phones) has taken about a transmutation to the world of digital devices. Applications running on these smartphone are popularly called mobile apps. Mobile apps allow app developers to enhance the functionalities of mobile devices. Currently there are over 2.5 million mobile apps across Apple iTunes store and Google Playstore(Statistica 2015). As per Nielsen (Nielsen 2014), in USA, on average people are spending over 11 hours with these mobile apps per month. This opens up a new avenue for businesses and brands to advertise their products and services. Web based advertisement has been a popular method for brands to reach target audiences. The annual revenues for web based advertisement in 2011, 2012 and 2013 were $31.7 billion, $36.57 billion and 42.78 billion respectively (IAB 2014). However, mobile advertising revenue soared a massive 92 percent to $19.3bn in 2013 from $10.1bn in 2012, confirming the adoption of mobile as an essential element of the marketer's toolkit (IAB 2013). The IAB report estimates that the revenue generated from mobile ads in 2011 was 3% of total revenue generated from all types of ads, 6% in 2012, and 16% till the second quarter of 2013. Various types of mobile phone advertisement exist, such as SMS advertisement, mobile browser display advertisement and advertisement in mobile apps. As users tend to spend more time with mobile apps than with anything else in mobile devices, the advertisement in mobile apps has become the most prominent form of mobile advertisement.

Though initial efforts on the mobile advertisement pivoted around extending browser based display advertisement framework and technology for mobile apps, mobile app advertisement has evolved to be fundamentally different than traditional display advertisement primarily for two reasons. First, the display advertisement is primarily based on cookie technology that allows users and their profiles to be identified from its browsing behavior. The absence of cookies in mobile apps and availability of several different types of device IDs (Aerserv 2014) make it hard for audience identification and user profiling in mobile advertisement. Second, in contrast to web-advertisement, the popularity of mobile app is a very transient phenomenon (Datta 2013). So the set of apps that are popular today do not remain so after a relatively short while (about 30 days). In display, the set of popular web sites are very static. These two reasons clearly make the mobile app advertisement to different from web-advertisement. In 2014, 76% of the display advertisement happens through direct relationships between media agencies and publishers (web sites) (Marvin 2014). This is possible because the users are pre-identified by cookies and the sites to target an advertisement can also be pre-determined based on their known popularity. However, none of these two are possible in mobile advertising. So, in mobile advertisement the agencies and publishers primarily resort to a system called programmatic advertisement buying that decides the placement of the advertisement during run-time. Though programmatic buying of media exists in display advertisement, the growth in programmatic buying is primarily due mobile app advertisement.

The programmatic advertisement system involves several stakeholders in the mobile advertisement ecosystem. Here instead of direct relationship between publishers (app publishers such as Roveo (Roveo 2015) and Kings (Kings 2015)) and agencies (media agencies such as Mindshare (Mindshare 2015) and Starcom (Starcom 2015)), publishers contact supplier side aggregators (called Supply Side Platform, SSP such as inMobi (InMobi 2015), Mopub (Mopub 2015)). When a request for advertisement placement arrives from a mobile device, the supply side aggregator consolidates such requests and contacts ad-exchanges or ad-networks such as Nexage (Nexage 2015) and OpenX (OpenX 2015). The ad exchange either delivers advertisements from campaigns that are already available to it or it gets the advertisement from agencies through an advertisement aggregator (called Demand Side Platform, DSP such as Millennial Media (Media 2015) and Fiksu (Fiksu 2015)). It should be noted that, currently, there are organizations (such as Mopub, Google Admob (Admob 2015)) serving all three roles--SSP, ad-exchange and DSP.

This paper is approached from the SSP perspective. One key decision that the SSP needs to take is the placement of advertisements; i.e., which advertisement to place in which available app, based on several criteria. One of the criteria is the audience and the relevance of the ads to the user. This is primarily done through location, device type and the IAB category in which the app belongs. For example, an advertisement may be targeted towards Samsung Galaxy users in Delhi who are using Entertainment app. However, multiple advertisement campaign will have overlapped target. So in spite of such filtering, at any point in time, the SSP will have several options in advertisement placement from various available campaigns. In such scenario, after the initial filtering of available campaigns based on target criteria (such as location, device type and IAB category), the SSP receives a set of candidate ads. The SSP needs to decide the advertisement placement based on parameters such as budget, duration of the campaign, target click through rate (CTR), target number of impressions and pacing (the speed at which the total budget will be spent). The key challenge in this decision process is that in online situations (i.e., when a bunch of requests for advertisement is received by a SSP), the final advertisement needs to be delivered by the SSP in under 100 milliseconds time (Internet Advertisement Bureau 2014). In this paper, we address this decision-making problem. We test the viability of an optimization-based online algorithm for this purpose.

We have made three specific contributions in the paper. First, we model the SSP side decision making problem in math programming form with the objective of maximizing SSP's revenue whereas meeting the objectives of campaigns in hand. Second, we describe a system that can take such decisions in a few milliseconds. The system is composed of an optimization problem, an offline rule based heuristic and an online algorithm. Third, with real life data sets from mobile advertising campaigns, we demonstrate some key features of both offline and online algorithm.

# 2 LITERATURE REVIEW

In digital online advertisement industry, programmatic ad placement has started to gain attractions after 2010, primarily with the proliferation of mobile advertisement. Research has, however, dealt with programmatic ad for a long time since the start of online advertisement at the beginning of this century.

The programmatic advertisement on display is primarily based on keywords associated with web sites. Google AdWords (AdWords 2015) with the acquisition of Doubleclick (Doubleclick 2015) is the pioneer in this field. In this framework keywords are extracted from the web sites' contents and publishers bid for keywords. Advertisement targeted to a set of keyword will be displayed in a web site, if the web site content also contains keywords from the targeted set (Hermann et al. 2005a, 2005b). Borgs et al.(2007) has developed an efficient keyword bidding approach for advertisers. There has been several other research works (Cary et al. 2007; Kitts and Leblanc 2004) in developing a bidding strategy for auctions in the keyword based advertisement. Though related, these are not applicable in mobile app advertisement, where keyword based advertisement does not work. In mobile apps, the contents are displayed in the app itself, so it is impossible for a third party system such as Google to identify the content and associate it with keywords. The mobile apps, such as games, are interactive in nature-- where the concept of keyword is not applicable.

Advertisement has been used to support the cost of internet-based online services (such as email like Hotmail or web sites like AOL) since the early days of the commercial internet. The earlier approach on scheduling advertisement involves rotating ad in web sites at pre-determined frequency and interval (Marsh & McAuliffe 1998). Kumar et al. (2006) demonstrated that this scheduling problem with the objective of maximizing the revenue of web site publisher is an NP hard problem. To maximize web site's advertisement based revenue an adaptive scheduling algorithm based on user clicks has been developed (Kumar et al. 2007). An advertisement scheduling algorithm for online browser based display has been developed to maximize the publisher's revenue (Roels and Fridgeirsdottir 2009). Past research on scheduling of online advertisement has also considered scenarios where a single web page has multiple ad placement slots and multiple advertisement needs to be placed in the web page as a package (Adler et al. 2002). More recent work on advertisement placement has incorporated audience targeting in the model and has applied artificial intelligence based technique to solve the scheduling problem (Deane 2012). Other variations of the same problem (i.e. scheduling advertisement in a web page), with various revenue models for publishers (such as cost per impression, cost per click) have been addressed in various past research (e.g., Amiri & Menon 2003, 2006).

In all the above display advertisement research, the advertisement scheduling has been in the ascendance of the publisher of the web site.The scheduling algorithms have been built up with the main aim of maximizing publisher's revenue. Contrary to these, in mobile the placement of the advertisement happens programmatically by the SSP. In case of mobile app advertisements, the app publishers use SSP API to handover the control of advertisement display to the SSP and the publisher keeps no control of such placement. The decision about which ad to place in which app and when is decided by the SSP alone, not by the publisher as has been addressed in past research related to display advertisement scheduling. In this context the scheduling algorithms developed in past research from the perspective of publishers have little applicability in mobile advertising.

To address this gap, in this paper, we develop the online scheduling algorithm for advertisement placement by SSP with the objective of maximizing the revenue of SSP (not publishers, as have been done in the past display advertisement scheduling literatures).

# 3 DESIGN SCIENCE APPROACH

We follow the design science approach (Hevner et al. 2004; Peffers et al. 2007), and elaborate upon four steps, namely*, Problem Statement and Motivation, Objective* (§3.1)*, Design Artifacts* (in §§3.2) and *Evaluation* (§4).

## 3.1 Stakeholder's Problem Motivation, Statement and Objective

The stakeholder addressed in the paper is the SSP. Given the increasing use of smartphones, the SSP has an enormous potential for revenue generation through appropriate ad-selection. Some issues compound this selection decision. First, there is a stream of ads that are coming to the SSP. For each of these ads, the time allowed to make a decision is in the order of milliseconds. Yet, the decisions taken must seek to improve the profits of the SSP. This motivates the need for providing both an optimal model and rules for making online decisions.

After due filtering based on relevance, the SSP is given a set of ads with known bids, impression price, CTR, CTR price, Conversion price, and Conversion rate, and cost of campaign. A fixed planning time-horizon, divided into periods, is given. Ads must be picked for display in smartphone apps. The goal is to maximize the total amount paid by the advertisers to the publishers over the time horizon. The constraints pertain to availability of the slots, pacing and exposure. If an ad is accepted and placed the SSP charges the ad-exchange (or the advertiser, whoever provides that ad) a fixed cost, which is less than or equal to the bid price given by the ad-exchange.

## 3.2 Artifacts

We develop three artifacts, namely, an offline optimal algorithm, an offline heuristic and an online algorithm (see Figure 1). The offline components are executed, say, on an hourly basis. Based on the solution of the optimal model, the accuracy of the rules in the rule-based heuristic is determined. If the rules are found adequate (i.e. accuracy is acceptable), then they are used as-is in the online algorithm; otherwise the set of rules will be modified for the online component. The online component is executed on the stream of ads; this is done on a real-time basis.

### *3.2.1 Artifact1: Optimal Model*

The optimization model for addressing the above problem is given below. The objective is to maximize the sum of the revenues from the impressions, CTRs, conversions and bids. The constraints are as follows:

- *Slot Constraint* indicates the number of slots available (constraint 2)
- *Pacing Constraint*. Advertisers need to manage their budgets within the periods, so that the budget expenditure is staggered (i.e., paced) across the periods within the planning horizon (constraint 3) (Facebook 2015).
- Exposure Constraint. Each ad-exchange specifies the number of impressions (constraint 4). This type of exposure requirements are also seen in real-life scenarios (Ad-Media 2015).

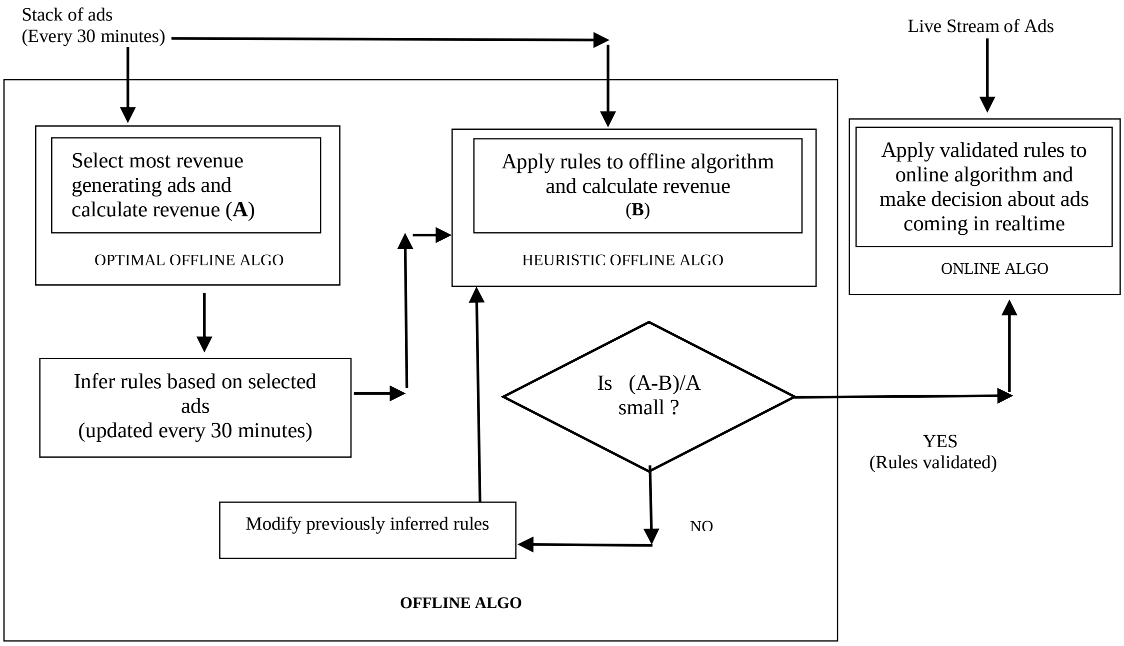


*Figure 1. System Artifact combining Online and Offline Algorithm*

Notations

*Input Variables*

| | |
|---|---|
| $n$ | *Index for ad-exchange platforms* |
| $a$ | *Index for advertisers* |
| $i$ | *Index for ads of a particular advertiser* |
| $j$ | *Index for ad-slots available at a particular instance* |
| $t$ | *Index for time* |
| $BID_{naijt}$ | *Bidding Price of $i^{th}$ ad of $a^{th}$ advertiser coming from ad-exchange n for $j^{th}$ ad-Slots at $t^{th}$ time* |
| $CONV_{naijt}$ | *Conversion rate of $i^{th}$ ad of $a^{th}$ advertiser coming from $n^{th}$ ad-exchange for $j^{th}$ ad-slot at $t^{th}$ time* |
| $CTR_{naijt}$ | *Click Through Rate of $i^{th}$ ad of $a^{th}$ advertiser coming from ad-exchange n for $j^{th}$ ad-slot at $t^{th}$ time* |
| $CTRPRICE_{naijt}$ | *CTR Price of $i^{th}$ ad of $a^{th}$ advertiser coming from ad-exchange n for $j^{th}$ ad-slot at $t^{th}$ time* |
| $CONVPRICE_{naijt}$ | *Conversion Price of $i^{th}$ ad of $a^{th}$ advertiser coming from ad-exchange n for $j^{th}$ ad-slot at $t^{th}$ time* |
| $COST_{naijt}$ | *Amount of money required to run the campaign of the corresponding ad* |
| $IMP_{naijt}$ | *Impression Price of $i^{th}$ ad of $a^{th}$ advertiser coming from ad-exchange n for $j^{th}$ ad-slot at $t^{th}$ time* |
| $BUDGET_{nat}$ | *Budget of $a^{th}$ advertiser of $n^{th}$ Ad-Exchange* |
| $REQIMP_n$ | *Total number of impressions required by nth ad-exchange* |
| $PERCENT_{nat}$ | *Pacing percentage of $a^{th}$ advertiser of nth ad-exchange at $t^{th}$ time instance* |
| $SLOT_t$ | *Number of Ad-Spaces available in Smartphone applications at $t^{th}$ time instance* |

*Decision Variables*

$X_{naijt}$ — *1, if $i^{th}$ ad of $a^{th}$ advertiser from nth ad-exchange wins the bid for being placed/pushed in jth ad-slot at $t^{th}$ time instance*
*0, otherwise*

*Objective*

$$\text{MAX} \quad \sum_n \sum_a \sum_i \sum_j \sum_t CTR_{naijt} CTRPRICE_{naijt} X_{naijt} + \sum_n \sum_a \sum_i \sum_j \sum_t IMP_{naijt} X_{naijt} + X_{naijt} \sum_n \sum_a \sum_i \sum_j \sum_t BID_{naijt} X_{naijt} + \sum_n \sum_a \sum_i \sum_j \sum_t CONV_{naijt} CONVPRICE_{naijt} X_{naijt} \tag{1}$$

SUBJECT TO CONSTRAINTS,

$$\sum_n \sum_a \sum_i \sum_j X_{naijt} \leq SLOT_t \quad , \forall_t \tag{2}$$

$$\sum_i \sum_j CONV_{naijt} CONVPRICE_{naijt} X_{naijt} + \sum_i \sum_j CTR_{naijt}\, CTRPRICE_{naijt} X_{naijt} + \sum_i \sum_j COST_{naijt} X_{naijt} + \sum_i \sum_j BID_{naijt} X_{naijt} + \sum_i \sum_j IMP_{naijt} X_{naijt} \leq BUDGET_{nat} PERCENT_{nat} \quad ,\forall_{n,a,t} \tag{3}$$

$$\sum_a \sum_i \sum_j \sum_t X_{naijt} \leq REQIMP_n \quad , \forall_n \tag{4}$$

$$X_{naijt} \in \{0,1\} \tag{5}$$

The above model seeks to allocate ads so as to maximize ad-revenue of the SSP. From a mathematical perspective, our problem is similar to the advertising scheduling problem in the web context, which is known to be NP-hard. Thus, based on this similarity to the web-ad scheduling problem, we are developing heuristics, although a formal proof would be necessary regarding the theoretical complexity of the model.

### 3.2.2 *Rule-based Offline Heuristic*

A key objective of our research is to schedule ads in real-time. This makes the optimization model not viable (note a viable solution, as mentioned earlier, must provide a decision in the order of tens of milliseconds). However, from the experience gained from solving many optimization instances, it would be possible to generate simple rules that are quick to execute yet provide a solution that are close enough to the optimal one. This section picks up on this idea.

First, we provisionally select ads that satisfy the thresholds (Step 1 of Algorithm1). Then total expense (x), revenue (y) of each ad is calculated and the entire set of ads is sorted according to descending order of revenue (steps 2 and 3 of Algorithm 1). This is because ads generating greater revenue should get higher priority than comparatively lower revenue generating ads, so as to increase the overall profit of the SSP. Next, all relevant criteria such as budget, availability of slots etc. are verified and ads are placed accordingly. Lastly, total revenue of all the selected ads from offline heuristics is compared with the revenue generated from mathematical model to judge the solution accuracy of the offline algorithm. Theoretical discussions of artifacts are explained in the later part of this section.

*Algorithm 1: Rule-based Offline Heuristic*

*Step1* *Provisionally select ad i meeting the following rule (see below for the method to get thresholds):*

*i.* $\frac{conv_i convprice_i}{ctr_i ctrprice_i}$ ≥*threshold1 ii.* $\frac{imp_i}{bid_i}$ ≥*threshold2 iii .*$\frac{conv_i}{ctr_i}$ ≥ *threshold3*

*Step 2* *For each selected ad i:*

$x = Ctr_i\, CtrPrice_i + Conv_i\, ConvPrice_i \ + \ ImpressionPrice_i + Bid_i \ + Cost_i$

$y = \ Ctr_i\, CtrPrice_i \ + \ Conv_i ConvPrice_i \ + ImpressionPrice_i + Bid_i$

*Step 3* *Sort ads selected in Step 1, according to descending order of y.*

*Step 4* *If (x<Budget) and (# of available slots>0) and (required no. of impressions>0):*
*Place ad in that schedule*
*Calculate total revenue generated from the whole dataset.*

*Step 5* *Check the percentage difference between Optimal Model and Offline Algorithm solution.*

*Algorithm 2: Threshold Computation for Offline Heuristic*

*Step 1* *Calculate* $U_i = \frac{conv_i convprice_i}{ctr_i ctrprice_i}$ , $V_i = \frac{imp_i}{bid_i}$ , $Z_i = \frac{conv_i}{ctr_i}$;
*[ where i= selected ads through optimal Model]*

*Step 2* *Select the minimum value of* $U_i$*,*$V_i$ *and* $Z_i$ *among all selected ads and those minimum values are set as thresholds or rules. So, threshold1= Minimum(*$U_i$*), threshold2= Minimum(*$V_i$*), threshold3= Minimum(*$Z_i$*);*

Intuitively, the steps (i.e., rules) of the heuristic mirror the objective function. Click Through Rate and conversion rate are important factors to judge ad's performance. Hence, the ratio of these two is calculated to find the allowable threshold for incoming ad. Similarly, ratio of product of CTR and CTR Price to Conv and Conv Price is calculated to check the minimum contribution to the revenue required by an ad for getting placed inside app. Lastly, ratio of imp to bid is calculated to find out the major contribution of an ad. These rules are applied to heuristics to select ads in real time.

### *3.2.3 Artifact 3: Online Heuristic*

The online algorithm is used to select individual ads from a stream of ads, based on the rules inferred from mathematical model and three other important criterions such as budget, slot availability, required number of impression. The key difference between the offline heuristic and the online counterpart is that in the offline version, all the ads are considered together. Specifically, this is done by collecting a set of ads and sorting them on some criteria (Step 3 of Algorithm 1 above), whereas decisions in the online algorithm are taken as the ads stream-in.

*If ad does not meet rules (Step 1 of offline algorithm):*
*Go to the next ad in the stream*
*Else If (x<Budget and (# of available slots>0) and (required no. of impressions>0):*
*Place ad in that slot, update budget, #. of slots and #. of impressions.*
*End if*

# 4 EVALUATION

To evaluate our proposed approach, we need to demonstrate that both the off-line rule-based heuristic and the online algorithm provides results that are acceptable compared to the optimal result. Though the final system is proposed as a series of optimal algorithm, off-line rule based heuristic and online algorithm – where each of the later stage is fed by the preceding stage, the accuracy of the complete system will depend on the accuracy of the rule based heuristic and online algorithm. In addition to the accuracy, for online algorithms the time taken to run is also very important. As per IAB, online determination of ad placement needs to happen within maximum 100 milliseconds (IAB 2012). We need to demonstrate whether the online component of our system meets that requirement.

### 4.1 Data

For the purpose of demonstrating our proposed approach, we have collected real-world data from few mobile advertisement companies and identified the campaign attributes that are relevant in this particular research. These attributes are,

*Imp Price*: Amount paid by the ad-exchange to the SSP per 1000 impressions of an ad (Imp 2015)
*Bid*: Amount that ad-exchange is willing to pay maximum for each ad-slot (Bid 2015)
*CTR*: Click through rate or total number of clicks/total number of impression (CTR 2015)
*CTR Price*: Money paid by the ad-exchange to the publisher for each click on ad (CTR Price 2015)
*Conversion Rate*: probability of purchasing from an advertisement (Conversion Rate 2015)
*Conversion Price*: Money paid by the advertiser to the publisher for each conversion (Price 2015)
*Cost*: Cost of setting up an advertisement campaign. This cost has been prorated in the duration in consideration (Cost 2015)

Table 1 provides a sample of real-world data-set on which we have run experiments. We randomly identified two sets of ads from this data set, with each ad-set (named S1 and S2) having 750 ads. We assume there are 25 ad-slots available at each time, for which the SSP needs to determine the placement of ads from its available advertisements (set S1 or S2), i.e. $SLOT_t = 25, \forall t \in T$.

| ID[1] | IMPPRICE | BID | CTR | CTRPRICE | CONVERSION RATE[2] | CONVERSION PRICE | COST |
|---|---|---|---|---|---|---|---|
| 12441 | 13.7 | 13 | 0.000289 | 19.20 | 0.000467893 | 75.30 | 9.0 |
| 21321 | 26.2 | 5 | 0.009483 | 88.20 | 0.000366619 | 96.60 | 6.0 |
| 21151 | 24.4 | 12 | 0.000698 | 25.70 | 0.000586800 | 172.6 | 9.0 |
| … | ... | ... | … | ... | … | … | ... |
| 13131 | 16.0 | 54 | 0.004867 | 4.60 | 0.000985934 | 14.10 | 7.0 |
| 13351 | 45.2 | 28 | 0.006210 | 91.80 | 0.000696230 | 83.40 | 10 |

*Table 1 Sample Real World Data Set[3] (Imp Price, Bid, CTR Price and Cost are measured in same units)*

### 4.2 Performance Metric

As discussed before we evaluate two aspects of our solution. (1) We evaluate the accuracy of heuristic and online algorithm and (2) we evaluate the time to run for online algorithm.

For accuracy, we measure the solution quality by percentage difference of the total revenue of SSP obtained through heuristic or online algorithm in comparison to the optimal value.

$$Percentage\ difference = 100 \times \frac{Opt - Apprx}{Opt},$$

Where *Opt* is the optimal revenue for SSP obtained by solving the math programming and *Apprx* is the revenue obtained by heuristic of online algorithm.

For time to run, we measure the total time to run the solution by clocking the beginning of the program and the end of the program.

### 4.3 Experiment Design

We conduct a $3 \times 2 \times 2$ experimental design by considering three levels of budgets (low, medium and high), two levels of pacing (uniform and varying), and two ad-sets (S1 and S2). This is shown in Table

[1]ID is the combination of Ad-Exchange ID, Advertiser ID, Ad ID, Ad-Slot ID, Time of arrival of bids in Table 1.
[2]Conversion Rate, Conversion Price are assumed to be significantly lower than CTR and higher than CTR price respectively
[3] Due to space constraint, small subset of original dataset is included in Table 1

2. To set the budget, we compute the sum of the top-25 ad-bids in the data-set (since there are 25 ad-slots at a time). Given this, the budgets for the $B_L$, $B_M$ and $B_H$ conditions are set to 30%, 60% and 90% of this total respectively.

| | | **Pacing** | | | |
|---|---|---|---|---|---|
| | | Uniform ($P_U$) | | Varying ($P_V$) | |
| **Budget** | Low ($B_L$) | $B_LP_US_1$, | $B_LP_US_2$^ | $B_LP_VS_1$, | $B_LP_VS_2$ |
| | Medium ($B_M$) | $B_MP_US_1$, | $B_MP_US_2$ | $B_MP_VS_1$, | $B_MP_VS_2$ |
| | High ($B_H$) | $B_HP_US_1$, | $B_HP_US_2$ | $B_HP_VS_1$, | $B_HP_VS_2$ |

^ Each experiment is repeated 10 times.

*Table 2. Experimental Design*

The uniform and varying pacing is implemented by providing a $\pm$0.5% and$\pm$9% variations of budget in each period. We conduct ten trials of each experiment, with each trial being obtained by generating ten sets of pacing numbers, according to the experimental condition (e.g., for $P_U$, the variations are generated within a 0.5% variation). Thus, our entire experiment consists of total 120 runs, with 750 ads being involved in each run.

# 5 EXPERIMENTAL RESULTS

## 5.1 Rule-based heuristic

Table 3 shows the percentage differences for the ten randomly selected trials, out of 120 total numbers of trials of the various experimental conditions. We notice that the percentage differences are very low (< 20%) in most of the cases ($B_LP_US_2$ is not reported since solutions from both optimal model and heuristic are zero owing to low budget, thus making the percentage difference zero in all trials.) A plot of select scenarios is shown in Figures 1. We plotted the distribution of percentage differences across all trials in Figure 2. Figure 3 shows that the error is less than 5% for about 60% of our problems, and less than 10% for 80% of problems. And, only for 5% of the problem instances the percentage difference for rule based heuristic is greater than 20% (note that the Figure 3 gives cumulative numbers).

To demonstrate the statistical significance of our experimental results, we did paired t-tests comparing the optimal solutions with the heuristic. The t-test showed that there is no significant difference between optimal and heuristic solution with p-values less than 0.001 in most of the cases where as difference exists in few cases with p-values greater than 0.001.

| Trial | $B_MP_U$ $S_1$ | $B_HP_V$ $S_1$ | $B_LP_U$ $S_1$ | $B_LP_V$ $S_1$ | $B_HP_V$ $S_1$ | $B_MP_V$ $S_1$ | $B_MP_U$ $S_2$ | $B_HP_U$ $S_2$ | $B_LP_V$ $S_2$ | $B_HP_V$ $S_2$ | $B_MP_V$ $S_1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 0.06 | 0.10 | 4.90 | 23.00 | 2.10 | 4.10 | 0.60 | 14.59 | 1.09 | 1.30 | 7.80 |
| 2. | 1.20 | 2.70 | 15.00 | 7.50 | 0.30 | 1.30 | 0.03 | 13.50 | 0.00 | 5.60 | 4.80 |
| 3. | 0.04 | 3.06 | 15.00 | 14.40 | 4.80 | 1.10 | 8.80 | 5.80 | 0.00 | 5.20 | 8.80 |
| 4. | 0.06 | 0.20 | 25.00 | 9.30 | 3.75 | 3.80 | 10.90 | 1.40 | 2.10 | 7.40 | 6.60 |
| 5. | 1.70 | 0.10 | 2.70 | 23.00 | 0.20 | 0.40 | 0.70 | 12.50 | 0.00 | 5.70 | 1.90 |
| 6. | 2.30 | 1.50 | 28.00 | 13.00 | 2.20 | 4.40 | 1.10 | 2.50 | 0.00 | 4.00 | 6.10 |
| 7. | 0.01 | 4.02 | 20.00 | 5.80 | 0.20 | 1.60 | 0.00 | 12.60 | 4.60 | 7.40 | 5.90 |
| 8. | 0.40 | 0.50 | 26.00 | 10.50 | 0.46 | 1.80 | 9.50 | 5.40 | 0.00 | 5.70 | 10.00 |
| 9. | 0.80 | 0.60 | 14.00 | 17.00 | 5.00 | 1.00 | 1.50 | 17.40 | 3.90 | 4.70 | 1.30 |
| 10. | 0.80 | 5.00 | 29.00 | 14.00 | 2.10 | 2.80 | 8.25 | 13.30 | 5.50 | 1.90 | 2.70 |

*Table 3. Percentage error of Heuristic Algorithm*

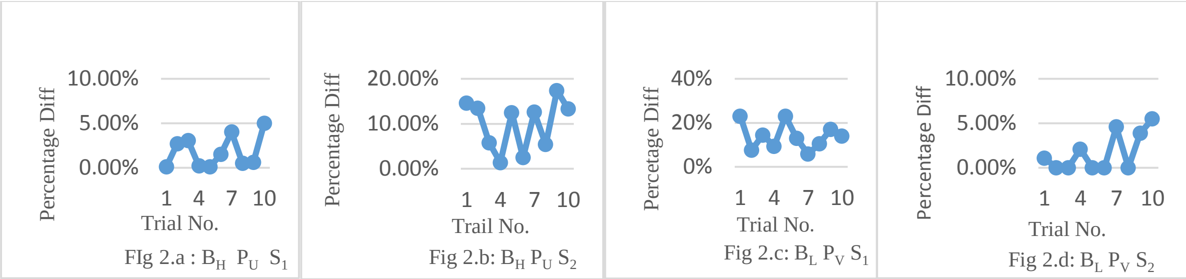


*Figure 2. Plot of Percentage difference of select trials for select experimental settings*

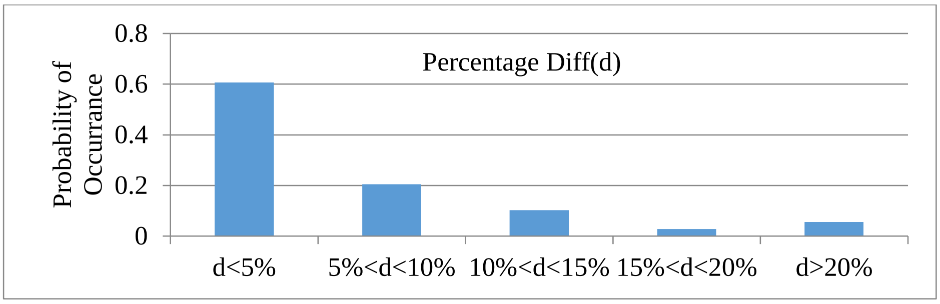


*Figure 3. Percentage Difference(d)*

We also computed the running time taken by the rule-based heuristic. On average the running-time was 10 seconds for 30 minutes campaign duration in consideration by SSP. Whereas the optimal problem is NP hard and can take easily several hours to come up with a solution for such campaign durations.

*The above result demonstrates that the rule based heuristic can provide a solution that is in most cases (80% of instances) within 10% of the optimal solution and the rule based heuristic is fast to apply at periodic intervals (e.g. every 30 min).*

### *5.2* Online Algorithm

Though the rule based heuristic can run within 10 seconds, it is impossible to apply it when stream of ad-requests from various mobile apps running in users' devices are arriving at SSP. As mentioned before the SSP needs to decide which ad to place in which slot within 100 milliseconds. Thus we need to resort to the online algorithm that can take the decision of ad placement as and when ad-requests arrive.

As proposed in Section 3, the online algorithm will work in conjunction with the rule-based heuristic, which runs in periodic interval. However, the accuracy of the complete system will be impacted by the accuracy of the online algorithm.

To demonstrate the accuracy of the online algorithm, we compare the revenue obtained by the online algorithm and the optimal algorithm with the computation of percentage differences. We ran the experiment as described in Section 4 (Experiment Design).

Due to space limitation we present the result of randomly selected 20 problems in Table 4. In case of the online algorithm, for about 50% of the problem instances, the percentage difference is less than 25% from the optimal result, and only for 10% of the problem instances the percentage difference is more than 40%.

|  | Mathematical Model | Real Time Algorithm | Percentage Difference |
|---|---|---|---|
| 1 | 1846.19 | 1386.21 | 24.92 |
| 2 | 5643.12 | 4232.31 | 25.00 |
| 3 | 3974.71 | 3295.73 | 17.08 |
| 4 | 2345.62 | 2016.69 | 14.02 |
| 5 | 2179.30 | 1833.02 | 15.89 |
| 6 | 3874.56 | 2218.97 | 42.73 |
| 7 | 4180.00 | 2218.97 | 46.91 |
| 8 | 3912.66 | 2637.52 | 32.59 |
| 9 | 4135.82 | 2769.05 | 33.05 |
| 10 | 1812.33 | 1456.50 | 19.63 |
| 11 | 1916.37 | 1409.80 | 26.43 |
| 12 | 4247.50 | 3292.79 | 22.48 |
| 13 | 4289.06 | 2854.95 | 33.44 |
| 14 | 4404.58 | 3292.37 | 25.25 |
| 15 | 4556.33 | 3556.23 | 21.95 |
| 16 | 3889.94 | 3119.42 | 19.81 |
| 17 | 3324.67 | 2887.44 | 13.15 |
| 18 | 3445.22 | 2334.56 | 32.24 |
| 19 | 4112.33 | 3342.89 | 18.71 |
| 20 | 3326.44 | 2234.55 | 32.82 |

*Table 4. Accuracy of the Online Algorithm*

This less optimal result for online algorithm is acceptable due to real-time performance of the online algorithm.

Figure 4 demonstrates the running times of select trials in select experimental condition. For each experimental setting and trial, we provide the worst-case, best-case and average-case running times for the online algorithm. For example, for trial 1, the worst-case time is slightly over 20 ms for the $B_MP_US_1$ condition and almost 30 ms for $B_MP_US_1$.

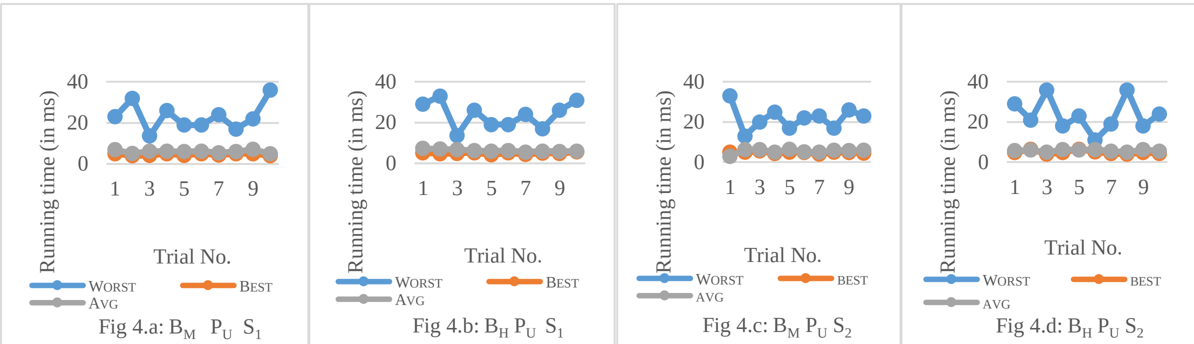


*Figure 4. Plot of running time of Online Algorithm for select experimental settings*

In our experiment, the running time for all trials in all experimental settings has been below 50 ms. The distribution of worst-case, best-case and average-case running time over ten trials across all 12 experimental settings are given in Figure 5. Figure 5 shows that the running time of our proposed online algorithm is much below stipulated 100 ms (as proposed by IAB).

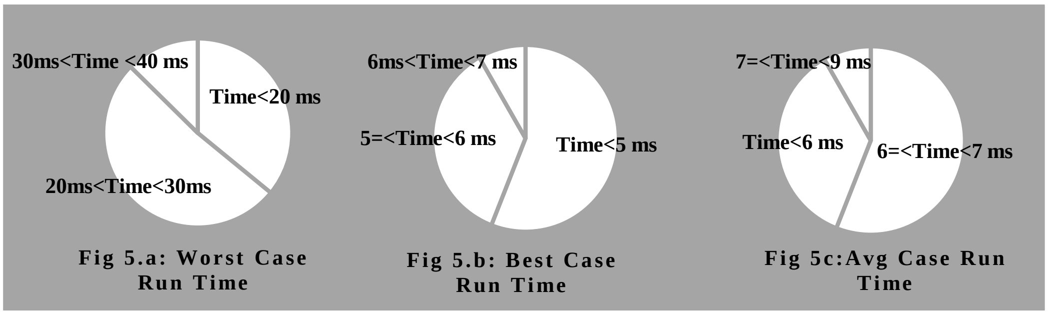


*Figure 5.* *Distribution of worst-case, best-case and average-case running time for online algorithm*

*Our experimental results on online algorithm demonstrates that though the accuracy of online algorithm is higher (below 25% in 80% of the cases), it can meet the real-time demand of ad placement in an SSP.*

# 6 IMPLICATIONS

## 6.1 Research Implications

As expected, the offline heuristic's accuracy is much higher than the online algorithm, however the running time for rule based heuristic is also several times more than the running time of online algorithm. The optimal model cannot be solved in reasonable time (few seconds) in most SSP instances. So it cannot be applicable for incoming stream of ad-requests in SSPs, although it can be applied periodically (e.g., once daily or weekly) on historical data to develop a broad-based solution which can be used to develop some rules. Given the dynamism of the mobile-advertisement, any solution based on historical data will not be fully accurate at run-time. So, the rules derived from optimal solution along with the rule based heuristic can be applied to derive ad placement at a more frequent periodic interval (e.g. every 30 min or 15 min). As demonstrated before, the solutions provided by rule based heuristic can enable SSP to achieve revenue, which is within 10% of the optimal. This is acceptable, although the rule based (offline) heuristic takes on average 10 seconds to run.

In an online advertisement system, the exact stream of ad requests arriving at SSP from mobile apps is unknown. Thus, the final decision about the ad placement needs to be taken at run-time when a stream of ad-requests are arriving at SSP. The maximum allowable time to take such decision is 100 ms, which is much higher than the running time of rule based heuristic. Thus rule-based heuristic is unsuitable for this purpose. The online algorithm presented in the paper can meet this running time requirement. However, the performance of the online algorithm comes at the cost of accuracy, which is increased to 25% compared to 10% in the case of rule-based heuristic.

Thus *we propose a system that is a combination of optimal model, rule-based heuristic and online algorithm – that meets the running time requirement and can also provide much accurate result.* The daily or weekly running of optimal model will generate a solution and a set of rules based on historical trend of incoming ad-requests stream. These rules along with rule based heuristic will generate a solution at every 15 min or 30 min, which is within 10% of optimal. This solution will then be further fine-tuned by the online algorithm as the ad-requests stream flows into the SSP system. The evaluation of this complete system both from the perspective of accuracy and running time is pending further investigation in this research.

Though ad-scheduling is a decade-old problem in display advertising, the scheduling problem has been tackled from the publishers' perspective. Whereas in mobile advertising, the advertisement scheduling and placement decision is taken by the SSP. This research work has the potential for providing a practical aid for SSPs in advertisement scheduling in mobile advertising.

### 6.2 Practical Implications for IS

The first practical implication is that the artifacts of our work can be used to develop a prototype of Intelligent Decision Support System (DSS) to demonstrate the application of IS in revenue maximization of SSPs in in-app advertisement placement. Since our results are based on real-life datasets (e.g., iPinYou.com), such a DSS will have practical implications. This initial phase of our research work can further be extended to another dimension by implementing more intelligent rule generation mechanism based on machine-learning techniques. Such robust rules can help integrate the whole system with virtual Ad-Exchange where Android/ smart phone applications are registered to sell available ad-spaces. Thus, we can develop a complete DSS to help Supply Side Platforms (app-developers) in mobile advertisement in an intelligent way. This makes an important contribution in the field of mobile advertising, as systems from the SSP’s perspective is almost scant in the literature, according to IAB report. Moreover, our results indicate the system will meet the target required for real-time implementation.

## 7 CONTRIBUTION, CONCLUSIONS AND FUTURE WORK

In this paper, we are given a stream of ads that have been screened for relevance with respect to an ad campaign. The objective is to decide on the ad-placement so as to maximize the revenue of an SSP. The decision has to be made in real-time; i.e., within under 100 milisecond. Three design artifacts have been developed for this purpose—an optimal algorithm, an offline heuristic and an online heuristic. Computational results indicate that the online heuristic provides a acceptable solution within a short time and can be periodically updated to keep in sync with the dynamic nature of mobile advertisement.

In the age of smartphone, application developers try to monetize their applications by selling ad-space to different advertisers. But, very few research has been done to fulfill app-developers’ need to earn money through in-app advertisement placement. In this paper we have included the mathematical model and heuristics to address these issues. Intelligent rule inferring mechanism can be included in the later stage of our work to make our system perform more efficiently. Two distinct contribution are made in this paper: (i) Integer Programming Model for revenue maximization of SSPs; (ii).Online algorithm for SSPs. In a later stage, we can develop an Intelligent DSS to integrate with the existing real-world advertising systems to help SSPs with revenue optimization.

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