The Firm’s Value and Timing of Adopting E-Commerce Using Real Options Analysis under Uncertainty

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Abstract

This paper applies real options analysis to analyze the optimal decision of adopting e-commerce when a firm’s output price is uncertain. The traditional discount cash flow method ignores the value of timing and operational flexibility. Therefore, it might underestimate the value of adopting e-commerce. The paper is the first study to build a theoretical derivation to detect the impact of adopting e-commerce on a firm’s value and timing based on the real options analysis. This study finds that increasing e-commerce investment and increasing price volatility will increase a firm’s value while delay the timing of adopting e-commerce. By contrast, decreasing efficiency of e-commerce will decrease a firm’s value and delay the timing of adopting e-commerce.

Keywords: Timing, E-commerce, Real options

1. Introduction

E-commerce has been developing very fast recently. Internet technology improves the flow efficiency of business supply chain and decreases production cost. It provides customers Internet order and increases sale performance. Efficiency improvements and cost savings already achieved through business-to-business. E-commerce have likely led to higher productivity growth, lower costs and reduced pricing power, which should allow a country to grow faster without inflationary pressures (Shim, 2000). On the other hand, it needs to invest tremendous amount of money in software and hardware. A company, hence, has to consider the cost and benefit before it implements e-commerce investment.

The impact of Information technology (IT) on firm performance has long been a subject of intense research, with issues studied ranging from measurement of the impact, to the conditions that are necessary to realize these impacts. However, researchers have pointed out these impacts are neither guaranteed upon implementation of the system nor are they uniform across the organization (Davern and Kauffman, 2000; Weill and Olson, 1989). Realization of the value of the system is conditional upon internal and external factors, some of which are controllable by the organization (Weill and Olson, 1989). Shaw (2003), thus, thinks that when millions of dollars of investments are being poured into e-business system projects in most larger companies, it is natural for IT managers to face the challenges of quantifying the value of e-business investments. It’s a pity that, up to now, there is still not more good methods to assess the value of e-commerce investments.

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As we study e-commerce and related topics in the financial field, many literatures are stressed on the performance after adopting e-commerce, information technology, and so on. The event study plays an important role in the related literature. It is an important methodology in management research that enables the assessment of the value created by firm initiatives, e.g. a firm’s announcement of its e-commerce initiative, based on the responses of the capital market to the announcement (Subramani and Walden, 2001). Since the early use of event study method in the information system literature (Santos, Peffers and Mauer, 1992), the use of this technique has recently surged. Researchers have employed it to investigate the market value implications of IT investments (Im, Dow and Grover, 2001), e-commerce initiatives (Subramani and Walden, 2001), creation of B2B e-marketplaces (Chen and Siems, 2001), and e-commerce sourcing (Agrawal, Kishore, and Rao, 2002). In the event study, however, it’s difficult to eliminate another factors’ impact on a firm’s value when event is happened. That is, the efficient market and “clean” data are asked in event study (Subramani and Walden, 2001).

The studies with regard to the subject of valuation and investment decision in e-commerce are likely rare. Subramaniam and Shaw (2002) study what is the value of B2B e-commerce to a buyer organization, how to measure this value, and what factors most affect the realization of the value of B2B e-commerce? They, however, are not investigating these issues under mathematical approaches. The tradition DCF model is easy to value a firm with e-commerce and to make a decision of adopting e-commerce. But the DCF method ignores the value of timing and operational flexibility (Dixit and Pindyck, 1994 and Trigeorgis, 1996). Using the real options theory to account for the value of future development is now a standard approach in finance among both academics and practitioners. Real options analysis is more focused on describing uncertainty and in particular the managerial flexibility inherited in many investments.

The real options approach was used in the field of natural resources, venture, and R&D investments. Recently, Billington, Johnson, and Triantis (2003) have explained some types of options in supply chain of high technology. In addition, Schwartz and Moon (2000) employ real options theory and capital budgeting techniques to the problem of valuing an Internet company. They report that the value of an Internet stock may be rational if growth rates in revenues are high enough. Taudes, Fuerstein, and Mild (2000) discuss the practical advantages of real options approach for the selection of a software platform. The real option model is used for deciding whether to continue employing SAP R/2 or to switch to SAP R/3. Benaroch, and Kauffman (1999, 2000) illustrate the value of applying real options analysis in the context of a case study involving the deployment of point-of-sale debit services by the Yankee 24 shared electronic banking network of New England. de Jong, Ribbers, and van der Zee (1999) concern the practical applicability of option pricing in valuing IT investments and compared with the NPV method. Kumar (1997) uses real option valuation to illustrate the value of improved responsiveness (i.e. flexibility) resulting from IT investments.

The literatures above demonstrate the importance of real options analysis in IT or e-commerce investment. They, however, don’t get certain solution on the timing of the e-commerce investment. When confronted with the availability of the investment of e-commerce, the firm have to choose not only if but also when to adopt e-commerce. Our model that investigates a firm’s value and timing of adopting e-commerce and get the closed form expression can help investors to make the optimal decision when confronted with the investment of e-commerce. By using real options model, this study attempts to find the impact of adopting e-commerce on a firm’s value under the assumption of uncertainty in a
firm’s output price. The paper is the first study to build a theoretical derivation to detect the impact of e-commerce adoption on a company’s value and timing based on the real options model.

The structure of the paper is as follows: Section 2 explains the assumptions and establishes a model about a firm’s value of adopting e-commerce under the Cobb-Douglas production function. In section 3, this study uses the model developed in section 2 to derive a closed form solution for a firm’s timing of adopting e-commerce. Extension of the model is investigated in Section 4. We study the impact of depreciation of e-commerce on a firm’s value and the timing of adopting e-commerce. In section 5, this study explores the sensitivity analysis of optimal e-commerce investment, analyzes the simulation results and draws some inferences by using numerical analysis. Section 6 sums up this paper with highlights of important findings.

2. Firm’s Value of Adopting E-Commerce

We assume a firm’s value, \( V(p_t) \), is determined by a stochastic output price \( p_t \). The e-commerce investment is irreversible, which implies that the firm’s value prior to the adoption of e-commerce includes an option to adopt e-commerce, whereas the firm’s value after adopting e-commerce does not. The firm’s value net of the option of adopting e-commerce is called the stand-alone value and is denoted by \( V_s(p_t) \). Hence, \( V(p_t) = V_s(p_t) + O(p_t) \), where \( O(p_t) \) denotes the endogenously determined value of the option to adopting e-commerce.

The output price \( p_t \) is assumed to follow the geometric Brownian motion:

\[
dp_t = \alpha p_t dt + \sigma p_t dz ,
\]

for constants \( \alpha < r , \sigma > 0 \). \( Z_t \) is a standard Brownian motion and \( r \) denotes the riskfree rate of interest. A firm’s instantaneous profit function is given by:

\[
p_t L^a K^b - w_t L - F ,
\]

where \( L \) denotes the variable production input (i.e. labor), whereas \( K \) is fixed (i.e. capital). The cost includes variable and fixed cost that is denoted by \( w_t L \) and \( F \) respectively. The cost per unit of input in \( L \) is denoted by \( w_t \). The output is determined by a Cobb-Douglas production function, \( L^a K^b \). The production function displays decreasing returns to scale with respect to the input (i.e. \( a, b < 1 \)). Then, the profit flow when the variable input, \( L \), is chosen optimally is given by:

\[
\pi(p_t) = \left( a^{1-a} - a^{1-a} \right) w_t^{1-a} p_t^{1-a} K^{1-a} - F ,
\]

\[
\equiv \Pi(w_t, a) p_t^{\gamma} K^{\gamma} - F ,
\]

where \( \gamma = \frac{1}{1-a} \) denotes elasticity (or power) of the price. Because \( a < 1 \), it follows that \( \Pi > 0 \) and \( \gamma > 1 \). Therefore, the profits are a convex increasing function of the output price, \( p_t \).

The stand-alone value of the firm, \( V_s \), can be seen as the discounted value of all future profits, and Dixit and Pindyck (1994) show that the stand-alone value of the firm is given by:
\[ V_s(p_t) = \frac{\Pi p_t^y K^{by}}{r - g(y)} - \frac{F}{r} = \psi p_t^y - \frac{F}{r}, \quad (4) \]

where \( g(y) = \gamma(r - \delta) + \frac{1}{2}\sigma^2 y(y - 1) \). For all this to make economic sense we need to assume \( r > g(y) \). We can recognize the expression for \( r - g(y) \) as just the negative of our fundamental quadratic evaluated at \( y \). Therefore by requiring that \( r > g(y) \), we are requiring \( Q(y) < 0 \). Then \( y \) must lie between the two roots of the quadratic \(^1\), specifically \( y < \beta \).

By taking account of production function of adopting e-commerce, the Cobb-Douglas production function (Goto and Suzuki, 1989) can be expanded as:

\[ h(L, E) = L(E)^{\tilde{a}} K^{\tilde{b}} E^c, \quad (5) \]

where \( E \) denote e-commerce investment and \( E > 0 \) under the situation of adopting e-commerce. We assume the effect of adopting e-commerce will spillover to the production department, so the production elasticity, \( a \) and \( b \), will become \( \tilde{a} \) and \( \tilde{b} \) respectively. Since the e-commerce will also affect labor input, we assume \( L(E) = L / E^d \). It is natural to assume that \( \tilde{a} \geq a \) and \( \tilde{b} \geq b \) because adopting e-commerce usually increases sales by Internet. One would probably expect adopting e-commerce to lead to a decrease in unit costs, but we don’t impose the restrictions on \( d \), which has to bigger than 0. This is because the labor input will increase to support e-commerce adoption in the beginning. As time goes on, the labor input will be saved when the efficiency of e-commerce developed. In addition, the influence of e-commerce on production function depends upon parameter \( c \). We assume \( 0 \leq c < 1 \), and equation (5) converges to original Cobb-Douglas production function when \( c \) and \( d \) is zero.

Variable cost function of adopting e-commerce is \( c(L, E) = w L / E^d \). Similarly, the profit flow, \( \pi_e(p_t) \), of adopting e-commerce when the variable input is chosen optimally is then given by:

\[ \pi_e(p_t) = \Pi(w_{L, \tilde{a}}) p_t^{\tilde{y}} K^{by} E^d - F, \quad (6) \]

where \( \Pi(w_{L, \tilde{a}}) = \left( a \frac{\tilde{a}}{\tilde{a} - a} \right)^{\frac{1}{1-a}} \), \( \tilde{y} = \frac{1}{1-a} \), and \( \theta = \frac{\tilde{a}(1-\tilde{a})d + c}{1-\tilde{a}} \). By using the same method we can derive the firm’s value of adopting e-commerce below:

**Proposition 1:** Assuming that investors are risk neutral and that there exists a risk free asset yielding a constant interest rate, \( r \), at which investors may borrow or lend freely, then the value of a firm to adopt e-commerce is given by:

\[ V_s(p_t) = \psi p_t^{\tilde{y}} E^d - \frac{F}{r}. \quad (7) \]

From the equation (7), we can find that the firm value is affected by the e-commerce investment through the parameter \( \theta \). The following differential results make it easier to understand:

\(^1\beta \) is positive root of quadratic equation and one can see Dixit and Pindyck, 1994 for further details.
\[ \frac{\partial V_\epsilon(p)}{\partial E} = \theta \psi p_\epsilon^{\gamma} E^{\theta-1}. \]  

(8)

The result is positive if \( \theta > 0 \). This implies that the adoption of e-commerce does not always enhance firm’s value. The firm’s value may decreases especially when labor inputs increase and efficiency of e-commerce doesn’t be developed in the initial stage.

The benefit of e-commerce is the difference of value before and after e-commerce adoption:

\[ V_\epsilon(p) - V(p) = \psi p_\epsilon^{\gamma} E^{\theta} - \psi p_\epsilon^{\gamma}. \]  

(9)

It is interesting to consider the special case where the production elasticity don’t change after adopting e-commerce, and hence \( \tilde{a} = a \) and \( \tilde{b} = b \). In this case the result of benefit is showed more simply by:

\[ V_\epsilon(p) - V(p) = \frac{\Pi p_\epsilon^{\gamma} K^{\by} (E^{\theta} - 1)}{r - g(\gamma)} = \psi p_\epsilon^{\gamma} (E^{\theta} - 1). \]  

(10)

From the special case above, one can realize the character of benefit from adopting e-commerce more easily. The benefit from adopting e-commerce is positive if \( \theta > 0 \) or consequently if \( \tilde{a}(1 - \tilde{a})d + c > 0 \). Since the benefit from adopting e-commerce is a convex increasing function of the output price, \( p \), it is procyclical: it rises at an increasing rate in economic booms and falls in economic busts. In considering e-commerce investment, a firm makes a tradeoff between the stochastic benefit and the cost of adopting e-commerce. As a firm has the option but not the obligation to adopt e-commerce, one will do so only when it is in one’s interest. This implies that the benefit from adopting e-commerce has call option characteristics, which is discussed in the next section.

3. The Timing of Adopting E-Commerce

When the value of the option which depends on the output price is large enough, the option will be exercised. That is, the firm’s decision to adopt e-commerce is contingent upon the output price. The firm’s value without adopting e-commerce, \( V(p) \), equals the sum of the stand-alone value plus the value of the option to adopt e-commerce. Thus,

\[ V(p) = \psi p_\epsilon^{\gamma} - \frac{F}{r} + O(p). \]  

(11)

The option value of e-commerce investment is denoted as follow:

\[ O(p) = \max [V_\epsilon(p) - V(p), 0]. \]  

(12)

The option value is worth to invest in the e-commerce only if the firm’s value with e-commerce is greater than that without e-commerce. We follow the steps of contingent claims valuation (Dixit and Pindyck, 1994 and McDonald and Siegel 1986), and the option, \( O(p) \), to invest in the e-commerce must satisfy the following differential equation:

\[ \frac{1}{2} \sigma^2 p^2 O'(p) + \mu p O(p) - r O = 0, \]  

subject to the boundary conditions:

\[ O(\bar{p}) = V_\epsilon(\bar{p}) - V(p) - E, \]
\[ O'(\bar{p}) = V_\epsilon'(\bar{p}) - V'(p), \]
\[ O(0) = 0. \]  

(14)

The first equation is the value-matching condition and it means that the value of the option must equal the net value obtained by exercising it. The second equation is the smooth-pasting
condition and that is, the graphs of \( O(p) \) and \( V_e(p) - V(p) - E \) should meet tangentially at \( \bar{p} \). Essentially, this condition ensures that \( \bar{p} \) is the trigger that maximizes the value of the option. The option value of e-commerce investment is gained in closed form expression below:

**Proposition 2:** Adopting e-commerce takes place the first when the state variable \( p \) hits the threshold \( \bar{p} \) from below. The value of firm's option to adopt e-commerce is given by:

\[
O(p, \bar{p}) = \left( \psi \bar{p}^\beta E^\theta - \psi \bar{p}^{\gamma} - E \right) \left( \frac{p}{\bar{p}} \right)^\theta \quad \text{for } p, \leq \bar{p}.
\]

The optimal threshold, \( \bar{p} \), of adopt e-commerce is the solution where

\[
\frac{\beta - \gamma}{\beta - \tilde{\gamma}} V(\bar{p}) + \frac{\beta}{\beta - \tilde{\gamma}} V_e(\bar{p}) = 1.
\]

In order to find some characters of adopting e-commerce, it is necessary to consider the special case where the production elasticity don’t change after adopting e-commerce, and hence \( \tilde{\alpha} = a \) and \( \tilde{b} = b \). In this case we get the firm’s option value in closed form expression below:

**Proposition 3:** Adopting e-commerce takes place the first when the state variable \( p \) hits the threshold \( \bar{p} \) from below. In the Special Case, i.e. \( \tilde{\alpha} = a \) and \( \tilde{b} = b \), the value of firm’s option to adopt e-commerce is given by:

\[
O(p, \bar{p}) = \left( \psi \bar{p}^\beta (E^\theta - 1) - E \right) \left( \frac{p}{\bar{p}} \right)^\theta \quad \text{for } p, \leq \bar{p}.
\]

The optimal threshold, \( \bar{p} \), of adopt e-commerce is given by:

\[
\bar{p} = \left( \frac{\beta E}{(\beta - \gamma) \psi (E^\theta - 1)} \right)^{\frac{1}{\gamma}}.
\]

The optimal strategy is to begin adopting e-commerce the first moment that \( p(t) \) equals or exceeds the trigger value \( \bar{p} \), as defined in equation (18). That is, the optimal timing to adopt e-commerce, \( T_E \), can be written as:

\[
T_E = \inf \left\{ t \geq 0 : p(t) \geq \left( \frac{\beta E}{(\beta - \gamma) \psi (E^\theta - 1)} \right)^{\frac{1}{\gamma}} \right\}.
\]

Under the dynamic situation that output price is stochastic, the higher the price the higher a firm profit is. The \( \bar{p} \) is the optimal threshold to adopting e-commerce. In other words, when the output price reaches the threshold \( \bar{p} \) from lower price, it’s the optimal timing to adopt e-commerce. Besides, we further express the price threshold equivalently in terms of a value threshold,

\[2 \text{ Since the factor } \frac{\beta}{\beta - \gamma} \text{ exceeds unity the price threshold exceeds the Marshallian break-even threshold. The firm’s net present value is therefore strictly positive. One can see Dixit and Pindyck, 1994 for further details.} \]
\[ V_p(\bar{p}) = \frac{\beta E^{\theta+1}}{(\beta - \gamma)(E^\theta - 1)}. \] (19)

It means that it's not the timing to adopt e-commerce unless the firm’s value has reached the threshold from the lower firm value.

The comparative static with respect to the merger threshold is very much in line with the predictions of real options theory. Namely, increasing uncertainty (\(\sigma\)) will lower \(\beta\) and therefore increase the (hysteresis) factor \(\beta((\beta - \gamma))\) and hence the threshold, \(\bar{p}\). Consequently, higher volatility delays investment, a well-known result from the real options literature (Dixit and Pindyck, 1994 and Trigeorgis, 1996). In addition, increasing volatility also raises the growth rate, \(g(\gamma)\), which in its turn speeds up investment. The other effect results from the convexity of profit in the output price and has been termed the Jensen’s Inequality effect on investment. For economically meaningful parameters, the former effect usually dominates. An increase in volatility therefore delays mergers.

The larger e-commerce costs usually delay the timing to adopt e-commerce. Differential threshold, \(\bar{p}\), by e-commerce investment, we can more understand the effect of e-commerce:

\[
\frac{\partial \bar{p}}{\partial E} = \frac{1}{\gamma} E^{\gamma-1} \left[ \frac{\beta}{(\beta - \gamma)(E^\theta - 1)^{\gamma}} \right] \left( \frac{E^\theta(1-\theta)-1}{E^\theta - 1} \right)
\]

is positive if \(\theta > 0\) and \(E^\theta(1-\theta) > 1\). Because \(\theta = (a(1-a) + c)/(1-a)\), the effect of e-commerce investment on threshold, \(\bar{p}\), depends on elasticity parameters of e-commerce, \(c\) and \(d\). When \(d\) is negative and \(c\) is small, the effect of e-commerce investment on \(\bar{p}\) is negative. This situation usually happened in the initial stage of adopting e-commerce. In the most time, the effect of e-commerce investment on \(\bar{p}\) is positive. It means that the higher investment of e-commerce will delay the timing of adopting e-commerce.

4. Depreciation Effect of E-Commerce
We have assumed thus far that the investment of e-commerce lasts forever. In reality, physical decay or technological obsolescence limits the development of e-commerce. Capital thereby depreciates through age, or use, or advance of competing technologies. One would expect that opportunity invest in a depreciating project of e-commerce would be less valuable.

We assume the form of depreciation according to the model of Dixit and Pindyck (1994). Exponential decay at any time \(T\), if the project of e-commerce has lasted that long, there is probability \(\lambda dT\) that it will die during the next short interval of time \(dT\). Now, from the initial perspective, the probability that the project dies before \(T\), or the cumulative probability distribution function of the random lifetime \(T\), is \(1 - e^{-\lambda T}\). The corresponding probability density function of \(T\) is \(\lambda e^{-\lambda T}\).

The benefit of e-commerce on cash flow can be denoted by \(\pi_e(p^*_t) - \pi(p^*_t)\). If the project of e-commerce lasts \(T\) years, the expected present value of its profit flows is

\[ E \int_0^T e^{-\mu t} [\pi_e(p_t) - \pi(p_t)] dt \]
By use the probability density of the lifetime for a Poisson process, we can obtain the expected value of a firm with depreciable e-commerce \( V_\lambda(p) \):

\[
V_\lambda(p) = \int_0^\infty \lambda e^{-\lambda T} \frac{\Pi p_t^\gamma K^{by} (E^\theta - 1)(1 - e^{-(r - g(\gamma))T})}{r - g(\gamma)} dT + V_s(p)
\]

\[
= \frac{\Pi p_t^\gamma K^{by} (E^\theta - 1)}{\lambda + r - g(\gamma)} + V_s(P)
\]

\[
\equiv \psi_\lambda p_t^\gamma (E^\theta - 1) + \psi p_t^\gamma - \frac{F}{r},
\]

where \( \psi_\lambda = \frac{\Pi K^{by}}{\lambda + r - g(\gamma)} \). The firm’s value with depreciable e-commerce compared with that in equation (7), the discount rate of firm value is increased by \( \lambda \). That is, the result of equation (7) is the case of \( \lambda = 0 \).

By using the same method, we can get the optimal timing of adopting e-commerce under the case of deprecation:

**Proposition 4**: Adopting e-commerce takes place the first when the state variable \( p_t \) hits the threshold \( \bar{p}_\lambda \) from below. In the Special Case, i.e. \( \tilde{\alpha} = a \) and \( \tilde{b} = b \), the value of firm’s option to adopt e-commerce under the case of depreciation is given by:

\[
O(p_t, \bar{p}_\lambda) = \psi_\lambda \bar{p}_\lambda (E^\theta - 1) - E \left( \frac{p_t}{\bar{p}_\lambda} \right)^\beta \quad \text{for} \quad p_t \leq \bar{p}_\lambda.
\]

The optimal threshold, \( \bar{p}_\lambda \), of adopt e-commerce under the case of depreciation is given by:

\[
\bar{p}_\lambda = \left( \frac{\beta E}{(\beta - \gamma)\psi_\lambda (E^\theta - 1)} \right)^{\frac{1}{\gamma}}.
\]

The threshold with depreciable case above is similar with that in equation (18).

5. Numerical Results

Although we derive the closed form solutions of firm value and threshold of adopting e-commerce, sensitivity analysis makes it easy to understand the inference of parameter to firm value and threshold under complicate calculations. In this section we perform some sensitivity analysis to exploiter the impact of parameter to a firm value and thresholds of adopting e-commerce. We set parameters as following: \( r = 0.05 \), return shortfall \( \delta = 0.045 \), \( \sigma = 0.1 \), \( a = 0.4 \), \( b = 0.55 \), \( c = 0.003 \), \( d = 0.001 \), \( w = 300 \), \( F = 5000 \), \( K = 1000000 \), and current output price \( p = 10 \).

Among settings of parameters, the setting value of \( c \) and \( d \) are much small than value of \( a \) and \( b \). This is because \( a \) and \( b \) are elasticity parameter of labor and capital respectively and labor and capital are essential factors of a firm. On the contrary, \( c \) and \( d \) are related elasticity parameters of e-commerce, and e-commerce isn’t essential factors of a firm although it becomes more and more important for a company. We thus set the value of \( a \) and \( b \) are much
larger than value of $c$ and $d$. One of course may set the value of $c$ and $d$ larger especially when efficiency of e-commerce has developed.

That the effects of e-commerce investments on thresholds are presented in figure 1. We can find the increasing e-commerce investments ($E$) increase the threshold and delay the timing of adopting e-commerce under both situations, considering ($\lambda \neq 0$) and not considering ($\lambda = 0$) depreciation. For general case ($\lambda = 0$), if e-commerce investment is 50 thousand dollars, it’s threshold to adopt e-commerce is 7.32. It thus means that now is the optimal timing to adopt e-commerce under the current price, 10. If the efficiency of e-commerce is not affected by the size of e-commerce investment, it isn’t suitable to adopt e-commerce immediately unless the e-commerce investment is under 68 thousand dollars (threshold is 10.02) for depreciable case with $\lambda = 0.01$ and the e-commerce investment is under about 45 thousand dollars (threshold is 9.94) for depreciable case with $\lambda = 0.03$ respectively.

That the effects of e-commerce investments on firm’s value are presented in figure 2. We can find the increasing e-commerce investment increase a firm’s value slowly under both general ($\lambda = 0$) and depreciation situations. Besides, depreciation lowers a firm’s value seriously and decreases a firm’s value as $\lambda$ increasing.

We can find the effects of price uncertainty ($\sigma$) on threshold in figure 3. The plot illustrates that increasing price uncertainty will increase the threshold and delay the timing of adopting e-commerce especially for depreciable case with $\lambda = 0.03$. The price threshold will increase rapidly when $\sigma$ becomes bigger. In the case of $\lambda = 0$, the adopting threshold, for instance, is
9.28 when $\sigma = 0.05$ and the threshold is 13.77 when $\sigma = 0.2$. This is consistent with the results of real option theorem (Dixit and Pindyck, 1994; McDonald and Siegel, 1986).

![Figure 3 The effect of $\sigma$ on thresholds of adopting e-commerce](image)

Figure 3 The effect of $\sigma$ on thresholds of adopting e-commerce

Figure 4 plots the joint effect of e-commerce elasticity $c$ and $d$ on the thresholds. The elasticity $c$ denotes the efficiency of e-commerce in the production output and $d$ denotes the efficiency of e-commerce in labor input. The larger the elasticity $c$, the efficient the e-commerce is, and therefore the threshold is getting lower when elasticity $d$ is given. But, when $c$ is large enough (e.g. $c = 0.005$), a firm can adopt e-commerce immediately in the range of $-0.003 < d < 0.003$. This is because adopting threshold is smaller than current price, 10 in the range of $-0.003 < d < 0.003$. Besides, the small the elasticity $d$, the higher the threshold of adopting e-commerce is especially when $d$ is negative and $c$ is small (i.e. $c = 0.001$).

![Figure 4 The effect of e-commerce elasticity $c$ and $d$ on thresholds](image)

Figure 4 The effect of e-commerce elasticity $c$ and $d$ on thresholds

6. Conclusions
This paper applies a real options approach to analyze the impact of adopting e-commerce on a firm’s value and timing when the output price is uncertain. By using a real option approach, this paper sheds some light on not only if but also when for a firm to adopt e-commerce. Under the condition of Cobb-Douglas production function, we derive a closed form solution for a firm’s value and the timing when e-commerce is adopted. By using numerical analysis to explore the sensitive analysis of optimal e-commerce investment, the results of simulation are as follows: (1) The larger the e-commerce investment, the higher the threshold of adopting e-commerce, and the greater a firm value is. (2) We find that increasing price volatility will increase the threshold, delay the timing of adopting e-commerce. This is consistent with the results of real option theorem. (3) Depreciation of e-commerce will make a firm’s value less valuable, increase the threshold, and delay the timing of adopting e-commerce.
References


